Nonlinear Models in Partial Differential Equations

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Talks of June 17th

(IN ELLABORATION)

On a conjecture by C. Sundberg: A numerical investigation

R. Glowinski & A. Quaini *

Carl Sundberg (University of Tennessee-Knoxville) conjectured some time ago that

$$\sup_{\varphi \in E} \frac{\displaystyle\int_0^1 \frac{|\varphi'|^4}{\varphi^6} dx}{1 + \displaystyle\int_0^1 |\varphi''|^2 dx} < +\infty \qquad \text{(SI)}$$

where

$$E = \{\varphi |\, \varphi \in H^2(0,1),\ \varphi(0) = \varphi(1),\ \varphi'(0) = \varphi'(1),\ \varphi \geq 1\}.$$

Our goal in this lecture is to report on the results of a numerical investigation that has been carried out these last few months in order to verify the veracity of the above Sundberg inequality. Indeed, our numerical experiments strongly suggest that (SI) is true and give also an approximation of the numerical value of the supremum over E of the functional in (SI). A brief description of the numerical methodology used to verify (SI) will be also provided; some of its features are reminiscent of an inverse power method for eigenvalue computations (which is not surprising since the above functional reminds of a Rayleigh quotient).

* Affiliation (Institute), Postal address. e–mail address@server

Fujita phenomenon in the hyperbolic space

Maria Assunta Pozio*

Since the pioneering work of Fujita in 1966, it is known that the Cauchy problem for reaction diffusion equations with power nonlinearities exhibits blow-up, but whether this is the case for all solutions, depends on a critical value, the Fujita exponent. We will discuss here similar results which hold in the hyperbolic space and were obtained with Catherine Bandle and Alberto Tesei. Such results show that the hyperbolic space is closer to a bounded domain, then to the whole Euclidean space.

*Sapienza-Università di Roma (Dipartimento di Matematica), P.le A. Moro, 2 - Roma 00161. pozio@mat.uniroma1.it

Finite Element Solution of Potential Flows Past Sails

A. Bermúdez, R. Rodríguez and M.L. Seoane *

In the last years the competition in nautical sports, as the America's Cup, has been the source of many important developments in mechanical engineering. The computer aided design is the key to the most efficient use of the wind force and to optimize the configuration of the hull and the sails in order to reach a greater speed (see, for instance, [4]).

This work deals with the mathematical and numerical analysis of a simplified two-dimensional model for the interaction between the wind and a sail. The wind is modeled as a steady irrotational plane flow past the sail, satisfying the Kutta-Joukowski condition. This condition guarantees that the flow is not singular at the trailing edge of the sail. The final aim of this research is to develop tools to compute the sail shape under the aerodynamic pressure exerted by the wind. This is the reason why we propose a fictitious domain formulation of the problem (see [3]), involving the wind velocity stream function and a Lagrange multiplier; the latter allows computing the force density exerted by the wind on the sail. Similar to [2], the Kutta-Joukowski condition is imposed in integral form as an additional constraint. The resulting problem is proved to be well posed under mild assumptions. For the numerical solution, we propose a finite element method based on piecewise linear continuous elements to approximate the stream function and piecewise constant ones for the Lagrange multiplier. Error estimates are proved for both quantities and a couple of numerical tests confirming the theoretical results are reported. Finally the method is used to determine the sail shape under the action of the wind. For this purpose, as a first step the sail is modelled as a linear string. The fluid-structure interaction problem is solved by using a segregated iterative algorithm.

References

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*Departamento de Matemática Aplicada, Universidade de Santiago de Compostela. Spain (alfredo.bermudez@usc.es, marialuisa.seoane@usc.es). Dep. de Ingeniería Matemática. Universidad de Concepción. Chile (rodolfo@ing-mat.udec.cl).

Stellerator model and some properties of the relative rearrangement

Jean-Michel Rakotoson *

The introduction of the Stellerator model in the mathematical literature by Ildefnso Diaz was a great opportunity to develop new properties for the relative rearrangement. We review some of those properties and we shall give new results related to the relative rearrangement.

*Laboratoire de Mathmatiques, SP2MI, Universit de Poitiers, Boulevard Marie et Pierre Curie, Tlport 2, BP30179, 86962 Futuroscope Chasseneuil Cedex, France. Jean-Michel.Rakotoson@math.univ-poitiers.fr

Traveling waves in a sawtoothed cylinder and their homogenization limit

Hiroshi Matano*

My talk is concerned with a curvature-dependent motion of plane curves in a two-dimensional cylinder with spatially undulating boundary. In other words, the boundary has many bumps and we assume that the bumps are aligned in a spatially recurrent manner.

The goal is to study how the average speed of the traveling wave depends on the geometry of the domain boundary. More specifically, we consider the homogenization problem as the boundary undulation becomes finer and finer, and determine the homogenization limit of the average speed and the limit profile of the traveling waves. Quite surprisingly, this homogenized speed depends only on the maximal opening angles of the domain boundary and no other geometrical features are relevant.

Next we consider the special case where the boundary undulation is quasi-periodic with m independent frequencies. We show that the rate of convergence to the homogenization limit depends on this number m.

This is joint work with Bendong Lou and Ken-Ichi Nakamura.

*University of Tokyo. e–mail matano@ms.u-tokyo.ac.jp