Nodal and multiple solutions for a prescribed mean curvature problem

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We present some existence and multiplicity results for *sign-changing* solutions of the nonlinear mean curvature problem

$$(P)\begin{cases} -\operatorname{div}\big(\nabla u/\sqrt{1+|\nabla u|^2}\big) = \lambda |u|^{p-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\lambda > 0$ and $2 < p < 2^*$. For instance, if the parameter λ is sufficiently large then (P) possesses at least one nodal solution u_λ with exactly two nodal domains. In addition, it is possible to prove the existence of arbitrarily many nodal solutions of (P), again for large values of λ.

Unlike many authors, we do not look for solutions of bounded variation. Instead we consider a perturbation of the degenerate part $1/\sqrt{1+s^2}$ and work in the Sobolev space $H_0^1(\Omega)$. This allows us to use classical variational techniques such as the Nehari manifold, the nodal Nehari set, and a symmetric version of the mountain pass theorem.

The above results have been obtained in collaboration with D. Bonheure (Brussels) and S. de Valeriola (Louvain-la-neuve).

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