On nodal domains of u solution to $-\Delta u = \mu u + f$, $u \in H_0^1(\Omega)$ and μ close to an eigenvalue

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Consider the Dirichlet problem

$$\begin{cases} -\Delta u = \mu u + f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

with Ω a smooth bounded domain in $I\!R^N$ and $f \in L^q(\Omega)$, $q > N$. Let $\hat{\lambda}$ be an eigenvalue of $-\Delta$ on $H_0^1(\Omega)$, with $\hat{\varphi}$ an associated eigenfunction. We compare the nodal domains of u with those of $\hat{\varphi}$. In particular, under suitable assumptions on f (in particular $\int_\Omega f\hat{\varphi} > 0$) and on the nodal domain of $\hat{\varphi}$, for μ sufficiently close to $\hat{\lambda}$, then the solution u of the problem has the same number of nodal domains as $\hat{\varphi}$, and the nodal domains of u appear as small deformations of those of $\hat{\varphi}$. But this is not always the case and we exhibit some examples and counterexamples for various hypotheses.

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