On a conjecture by C. Sundberg: A numerical investigation

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Carl Sundberg (University of Tennessee-Knoxville) conjectured some time ago that

$$\sup_{\varphi \in E} \frac{\int_0^1 \frac{|\varphi'|^4}{\varphi^6} dx}{1 + \int_0^1 |\varphi''|^2 dx} < +\infty \tag{SI}$$

where

$$E = \{ \varphi | \varphi \in H^2(0,1), \ \varphi(0) = \varphi(1), \ \varphi'(0) = \varphi'(1), \ \varphi \ge 1 \}.$$

Our goal in this lecture is to report on the results of a numerical investigation that has been carried out these last few months in order to verify the veracity of the above Sundberg inequality. Indeed, our numerical experiments strongly suggest that (SI) is true and give also an approximation of the numerical value of the supremum over E of the functional in (SI). A brief description of the numerical methodology used to verify (SI) will be also provided; some of its features are reminiscent of an inverse power method for eigenvalue computations (which is not surprising since the above functional reminds of a Rayleigh quotient).

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