Energy Balance Climate Models with Bio-Feedback

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Professor Díaz has made fundamental contributions to the study of energy balance climate models. Some of them serve as starting point for the topic of this talk, incorporating a bio-feedback in a 2-dimensional energy balance climate model.

It is well-known that there is a multitude of interactions between the climate system and the biosphere. E.g., the rise of temperature accompanied by increased rainfall may lead to a richer vegetation which in turn causes an higher absorption of solar radiation and therefore a further increase in temperature. On the other hand, higher temperatures associated with drier conditions may accelerate desertification and lead to a higher albedo with lower average temperatures as consequence.

Energy balance climate models describe the evolution of a long-term mean of temperature by employing the relevant balance equations for the heat fluxes involved. The horizontal heat flux is parameterized by a diffusion operator, and a bio-feedback can, e.g., be introduced by a Volterra map $V = V(u)\phi$ which is, say, the solution family (climate indicator u as parameter, ϕ a fixed initial vegetation) of a two-species competition system. A typical example for the resulting reaction-diffusion problem is

$$\begin{cases} c(x)\partial_t u - \nabla \cdot [k(x)\,|\nabla u|^{p-2}\,\nabla u] + g(u, V(u)\phi)(t)) \\ \qquad\qquad \in F(t, x, u, \overline{u}, V(u)\phi)(t)) \qquad t > 0,\ x \in M, \\ \overline{u}(t,x) := \int_{-T}^{0} \beta(s,x) u(t+s,x)\,ds,\ t > 0,\ x \in M, \\ u(s,x) = u_0(s,x), \qquad\qquad -T \le s \le 0, x \in M. \end{cases}$$

One is interested in nonnegative solutions $u = u(t,x)$ (temperature in Kelvin). M is a closed, compact, oriented Riemannian surface representing the Earth's surface, the positive functions c and k represent the thermal inertia and the diffusivity of the system, respectively, F stands for the absorbed solar radiation flux, and g represents the emitted terrestrial radiation flux.

I plan to discuss some models for V and the basic dynamics of the resulting problem.

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