A variance-expected compliance approach for topology optimization

Miguel Carrasco [*], Benjamin Ivorra [†], Rodrigo Lecaros [‡], Angel M. Ramos [†]

We consider an elastic homogeneous body $\Omega \subseteq \mathbb{R}^d$. We impose some support conditions in $\Gamma_u \subseteq \partial\Omega$,. We apply some external load forces to this body. Our purpose is to find the optimal distribution of material in Ω when the external load force has a stochastic behavior [1].

Assuming the linear response of the body material, the displacements is obtained by solving:

$$-\operatorname{div}(K\,e(u)) = f \text{ in } \Omega; u = 0 \text{ on } \Gamma_u; (K\,e(u)) \cdot n = 0 \text{ on } \partial\Omega \setminus \Gamma_u \cup \Gamma_t; (K\,e(u)) \cdot n = g \text{ in } \Gamma_t, \quad (1)$$

where $\Gamma_u \cap \Gamma_t = \varnothing$, f corresponds to an external load, g is a surface force applied to Γ_t, $u\colon \Omega \to \mathbb{R}^d$ is the vector of displacements, $e(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ denotes the strain tensor and $K = (K_{i,j,k,l})$ is the elasticity tensor. Under suitable conditions, the solution of (1) is unique [2].

We assume that K depends on a parameter $\lambda \in \Lambda$ measuring the amount of material at each point of Ω, Λ is the set of admissible material distribution, and $g \equiv 0$. The minimum compliance design problem can be stated as following [2]:

$$\min\{l(u(\lambda)) \mid \lambda \in \Lambda; A[\lambda](u(\lambda), v) = l(v), \text{for all } v \in \hat{H}\}, \quad (2)$$

where $A[\lambda](u,v) = \int_\Omega K_{i,j,k,l} e_{i,j}(u) e_{i,j}(v)\,\mathrm{d}x$, $l(u) = \int_\Omega f \cdot u\,\mathrm{d}x$, $\hat{H} = \{u \in [H^1(\Omega)]^d \mid u_i|_{\Gamma_u} = 0,\, i = 1, \ldots, d\}$ and $u(\lambda)$ denotes the unique weak solution of (1).

We consider that f is randomly perturbed by the random vector ξ (with $\mathbb{E}(\xi) = 0$). Thus, a stochastic topology design problem can be stated as [3]

$$\min\{\alpha \mathbb{E}[\Psi(\xi,\lambda)] + \beta \operatorname{Var}[\Psi(\xi,\lambda)] \mid \lambda \in \Lambda\}, \quad (3)$$

where $\alpha, \beta \geq 0$, $\alpha + \beta > 0$, $\mathbb{E}(\cdot)$ and $\operatorname{Var}(\cdot)$ are the expected value and the variance of the corresponding random function, respectively, and $\Psi(.,.)$ is defined by

$$\Psi(\xi,\lambda) = \quad \{\textstyle\int_\Omega (f+\xi) \cdot u\,\mathrm{d}x \mid u \in \hat{H}, \text{ s.t.: } A[\lambda](u,v) = \int_\Omega (f+\xi) \cdot v\,\mathrm{d}x, \text{ for all } v \text{ in } \hat{H}\}. \quad (4)$$

In this work, we use previous results obtained for truss structures [1, 3] to give an explicit expression of (3) and solve it by using a global optimization approach [4]. Then, we show that a good choice of (α, β) allows to obtain a material distribution of the body Ω stable to the considered load perturbations.

References

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[*] Facultad de Ingeniería, Universidad de los Andes, Chile: migucarr@uandes.cl

[†] Universidad Complutense de Madrid & Insituto de Matemática Interdisciplinar, Spain: ivorra@mat.ucm.es; angel@mat.ucm.es

[‡] Departamento de Ingeniería Matemática, Facultad de Ingeniería, Universidad de Chile, Chile: rlecaros@dim.uchile.cl