A parabolic-elliptic system of drift-diffusion type with subcritical mass in $\mathbb{R}^2$

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We consider the Cauchy problem to the following parabolic-elliptic system in $\mathbb{R}^2$:

$$\text{(CP)}\quad \begin{cases} \partial_t u = \Delta u - \nabla \cdot (u \nabla \psi), & t > 0,\ x \in \mathbb{R}^2, \\ -\Delta \psi = u, & t > 0,\ x \in \mathbb{R}^2, \\ u(0,x) = u_0(x), & x \in \mathbb{R}^2. \end{cases}$$

Here ψ is specified as

$$\psi(t,x) = \frac{1}{2\pi} \int_{\mathbb{R}^2} \log \frac{1}{|x-y|} u(t,y)\, dy,$$

and $u(t,x)$ and $u_0(x)$ are nonnegative. This system is a simplified version of chemotaxis system derived from the original Keller-Segel model in $\mathbb{R}^2$, and also a model of self-attracting particles in $\mathbb{R}^2$.

The total mass of the nonnegative solution u to (CP) is conserved, namely $\int_{\mathbb{R}^2} u(t,x)\, dx = \int_{\mathbb{R}^2} u_0(x)\, dx$, and the global existence of the solution heavily depends on the total mass $\int_{\mathbb{R}^2} u_0(x)\, dx$. It is known that the nonnegative solution may blowup in finite time in the supercritical case $\int_{\mathbb{R}^2} u_0(x)\, dx > 8\pi$. In this talk, we consider the subcritical case $\int_{\mathbb{R}^2} u_0(x)\, dx < 8\pi$ and discuss the global existence and large time behavior of the nonnegative solution.

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