Mathematical analysis of a thermoelastic problem

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In this work we carry out a mathematical analysis of the coupling between the motion and energy conservation equations for thermoelastic materials. Specifically, the existence and uniqueness of a quasistatic problem with mixed displacement-traction and Robin boundary conditions is obtained.

In the literature, there exist several existence results for thermoelastic problems, although not many deal with quasistatic problems with Robin boundary condition. A similar problem was studied by Viaño in [1], and by Figueiredo and Trabucho in [2] considering the dynamic motion equation. We use the same methodology in order to prove the existence of a solution of the problem, although we study the quasistatic problem without contact and for thermoelastic materials, taking into account that our reference temperature depends on the material point, our stress tensor has a particular thermal part and we consider a Robin boundary condition for the temperature. These contributions cause some difficulties to calculate *a priori* estimates. Furthermore, in order to prove the uniqueness their technique was not useful to us and the result is obtained following the methodology of Gawinecki [3].

Let $\Omega \subset \mathbb{R}^n$ (with $n = 2, 3$) be an open and bounded set with smooth boundary and let $[0, t_f]$ be the time interval of interest. Then, the problem we are going to study is finding a displacement field $\mathbf{u}(p, t)$ and a temperature field $\theta(p, t)$ in $\Omega \times (0, t_f]$, verifying:

- the equilibrium equations in $\Omega \times (0, t_f]$

$-\mathrm{Div}\,\boldsymbol{\sigma}(\theta, \mathbf{u}) = \mathbf{b}$	$\boldsymbol{\sigma}(\theta, \mathbf{u}) = \Lambda^{-1} : \boldsymbol{\varepsilon}(\mathbf{u}) - \alpha(\theta - \theta_r)(3\lambda + 2\mu)\mathbf{I}$
	Λ^{-1}: The inverse elasticity tensorial function.
	$\boldsymbol{\varepsilon}(\mathbf{u})$: The deformation tensor.
	α: The coefficient of thermal expansion.
	θ_r, $\mathbf{b}$: The reference temperature and the body forces.
	λ, μ: The Lamé's parameters.
$\rho_0 c_F \dot{\theta} = -\theta_r \alpha(3\lambda + 2\mu)\mathrm{Div}\ \dot{\mathbf{u}} + \mathrm{Div}\ (k\nabla\theta) + f$	ρ_0: The reference density.
	c_F: The specific heat at constant deformation.
	k: The thermal conductivity of the material.
	f: The body heat.

- the mixed displacement-traction conditions and Dirichlet, Neumann and Robin boundary conditions in temperature.

Under appropriate assumptions we prove the existence and uniqueness of solution using the Galerkin's method.

References

[1] Viaño Rey, J.M. *Existencia y aproximacin de soluciones en termoelasticidad y elastoplasticidad.* PhD thesis, Department of Applied Mathematics, Universidade de Santiago de Compostela, 1981.

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[3] J. Gawinecki. Existence, uniqueness and regularity of the first boundary-initial value problem for thermal stresses equations of classical and generalized thermomechanics. *J. Tech. Phys.*, 24(4):467–479 (1984), 1983.

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