Nonlinear models in partial differential equations arising in nuclear fusion

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We carry out mathematical analysis of some bidimensional nonlinear problems satisfied by the averaged poloidal flux of the magnetic field in the magnetic confinement of a plasma in the nuclear fusion. We show some mathematical models related to the stationary and evolution regime of a plasma in Tokamak and Stellarotor devices. The models can be formulated as an inverse problems since several nonlinear terms of the partial differential equation are not a priori known (non local terms). Using the current balance within each flux magnetic and the notion of relative rearrangement we can reformulate the problem as a non local one but having a direct formulation. We review some models concerning to Stellarator devices and we review some results about existence, uniqueness and regularity of solutions.

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