A variational approach to Navier-Stokes

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We introduce a variational approach to treat the Navier-Stokes equations both in dimensions 2 and 3. Though the method allows the full treatment in dimension 2, we seek to precisely stress where it breaks down for dimension 3. The basic feature of the procedure is to look directly for strong solutions, by minimizing a suitable error functional that measures the departure of feasible fields from being a solution of the problem. By considering the divergence-free property as part of feasibility, we are able to avoid the explicit analysis of the pressure. Two main points in our analysis are:
1. Coercivity for the error functional is achieved by looking at scaling.
2. Zero is the only critical value: global minimizers of the error are shown to have zero error (and thus they are solutions of the problem) by looking at optimality conditions, which lead to investigate the linearized problem.

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