

The Partial Differential Equations of Finance

Olivier Pironneau*

The object of financial mathematics is to price financial object and evaluate the risks.

To minimize risks self financing portfolios has been a great source of models. Very simple - yet unrealistic - assumptions such as brownian randomness and market completeness (no arbitrage) lead to stochastic differential equations and Itô calculus then gives the partial differential equations for pricing the financial objects.

Even in this restricted setting a large class of partial differential equations can be generated, mostly parabolic in nature, linear but with non constant coefficients. Variational methods work in a slightly adapted Hilbert space setting (weighted Sobolev spaces).

At first one dimensional in space, the partial differential equations are now multidimensional because of stochastic volatility models or because of stochastic interest rates or because the object depends on several assets. For these existence is not always guaranteed, the so called Feller condition is necessary in the case of Stein-Stein models. They can also be degenerate as in the case of Asian options.

New boundary conditions are needed sometimes such a for Bermuda options.

Non-linear systems are not used often so far; but an important class of objects do need nonlinear models, when early exercise is allowed such as for American options. Then the problem is best solved as a variational inequality.

When the Brownian assumption is relaxed, like when jump processes are added, a partial integro differential system is obtained, similar to the equations for radiative transfers in engineering. In several dimensions these problems are challenging and not solved yet.

Finally options on compound objects or basket options lead to partial differential equations in many dimensions as much as 50 or more. These are numerically extremely challenging.

*University of Paris VI, Laboratoire J.-L. Lions, Boite 187, Place Jussieu, F-75252 Paris 05. e-mail Olivier.Pironneau@upmc.fr