Competition of subspecies and structural stability. Survival of the best adapted or coexistence?

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Equations of population dynamics allow us to consider the competition of two species in an environment. According to the specific properties of their interaction, competition may lead to extinction of one of the species, but also in certain cases to coexistence of both.

In our work, we start from the dynamics of the population of a species; we split it into the populations of two subspecies (of the same specie) with the same properties. It appears that the corresponding dynamical system is structurally unstable, $i.e.$ small perturbations modify drastically the structure of the phase portrait of the dynamics. In that context, we introduce a small perturbation of the properties of one of the subspecies ("mutation"). It clearly appears that, according to the instability of the starting state, the issue of he mutation depends drastically on the very shape of the (even very small) perturbation. Specifically, when the modification may be clearly understood (whatever the circumstances) as an "advantage" of one of the subspecies on the other, the dynamics leads, generally speaking, to the extinction of the less adapted subspecies. But, *when the modification is "rather complex", containing features which are "advantages" or "disadvantages" according to the circumstances (in particular initial conditions), the dynamics leads generally to another equilibrium with coexistence of both subspecies*.

Analogous results follow from a starting situation mainly described by two species interacting in a predator / prey framework with a stable periodic cycle. The splitting of one of the species into two subspecies constitutes a structurally unstable system, and the perturbation (mutation) of one of the subspecies may have very different issues. In "rather complex" situations, the dynamics often leads to one or several periodic cycles with coexistence of the three new species.

On account of the fact that genetic mutations imply modifications *of a specific protein*, which, in its turn has in general an influence on *several behavioral properties*, it clearly appears that *the population dynamics lead to the extinction of the less adapted only in very elementary cases; in usual, more complex cases, it rather leads to the preservation of the variety*.

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