Mathematical Treatment of a climatological model with deep ocean effect

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This poster is a review of some results obtained in collaboration with J.I. Díaz in the last years. We have studied the existence and multiplicity of solutions to several transient and equilibria simple climate models.

In the last decades, many authors have studied the so-called global climate energy balance models (EBM) dealing with the evolution of the mean surface temperature of the Earth. Among them we can mention D. Arcoya, R. Bermejo, J.I. Díaz, M. Ghil and S. Childress, J. Hernández, G. Hetzer, G.R. North, B. Schmidt, X. Xu, etc.

In many of the previous models, the effect of the oceans is only considered in an implicit and empirical way in the spatial dependence of the coefficients. However, some works about the rapid climatic change in Glacial-Holocene transition (see Berger et al) show that it could be related to the past changes in deep water formation. In this work we study a model including the effect of the deep ocean based on the model proposed by Watts - Morantine.

The simplified model represents the evolution of the temperature U in a global ocean Ω with constant depth H. The upper boundary of Ω simulates the Earth surface. The governing equation for the ocean interior is parabolic and the upper boundary condition comes from an energy balance for the mean surface temperature of the Earth. In such energy balance, the absorbed energy depends on the planetary coalbedo β (which is discontinuous on U). The coalbedo is treated in the general class of multivalued graphs. More precisely, we shall assume that β is a bounded maximal monotone graph of $\mathbb{R}^2$. Other nonlinearity at the boundary concerns the surface diffusion proposed by P.H. Stone: its diffusion coefficient depends on the temperature gradient in order to include the negative feedback of the eddy fluxes. So, the energy balance involves in this way the p-Laplacian surface operator.

This kind of models is very sensitive to small fluctuations of Solar and terrestrial parameters. We also analyze how the Solar constant is related to the number of equilibrium solutions. One of the main difficulties in the mathematical treatment of these models comes from the presence of a nonlinear dynamic and diffusive boundary condition in its formulation.

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