Nonlinear fractional diffusion equations of porous medium type

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The study of diffusion involving fractional Laplacian operators is a topic of much current research. We present two models for flow in porous media including nonlocal (long-range) diffusion effects of such type.

The first one is based on Darcys law and the pressure is related to the density by an inverse fractional Laplacian operator. We prove existence of solutions that propagate with finite speed, which is unexpected in fractional diffusion models. The model has also the very interesting property that mass preserving self-similar solutions can be found by solving an elliptic obstacle problem with fractional Laplacian for the pair pressure-density. We use entropy methods to show that the asymptotic behaviour is described after renormalization by these solutions which play the role of the Barenblatt profiles of the standard porous medium model.

The second model comes from statistical mechanics considerations, generalizes the well-known linear fractional heat equation, generates a nice nonlinear contractive semigroup, and has infinite speed of propagation for all powers of the nonlinearity.

Some references

L. Caffarelli-J.L. Vazquez: arXiv:1001.0410 [math.AP] & arXiv:1004.1096 [math.AP]
A. de Pablo-F. Quiros-A. Rodriguez-J.L. Vazquez: arXiv:1001.2383 [math.AP] & arXiv:1104.0306 [math.AP]

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