Localization properties of solutions of parabolic equations with variable anisotropic nonlinearity

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We are interested in the character of propagation of disturbances from the data in solutions of parabolic equations with variable and anisotropic nonlinearity. The prototype of such equations is furnished by the equation

$$u_t = \sum_{i=1}^{n} D_i \left(|D_i u|^{p_i(x,t)-2} D_i u \right) + c_0 |u|^{\sigma(x,t)-2} u + f.$$

Anisotropy and variable nonlinearity lead to certain properties intrinsic for the solutions of equations of this type. We prove that unlike the case of isotropic diffusion the solutions vanish in a finite time even in the absence of absorption (i.e. if $c_0 = 0$), provided that the diffusion is fast in only one direction. It is shown that in the case of slow anisotropic diffusion the supports of solutions display a behavior typical for the solutions of equations with strong absorption terms: the support does not expand in the direction corresponding to the slowest diffusion. For certain ranges of the nonlinearity exponents the supports are localized both in space and time. We also discuss the influence of anisotropy on the blow-up of solutions and show that for equations with variable nonlinearity the effects of finite time vanishing and blow-up may happen even if the equation becomes linear as $t \to \infty$. The results were obtained in collaboration with S. Antontsev. The presentation follows the papers [2, 3, 4]; the main tool is the method of local energy estimates developed in [1].

The work was supported by the Research Project MTM2010-18427 MICINN, Spain

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