Équations aux dérivées partielles/Partial Differential Equations

Nonlinear elliptic boundary-value problems in unbounded domains and the asymptotic behaviour of its solutions

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Abstract — We use energy methods for the study of nonlinear boundary value problems with a superlinear term (e.g. \( g(x,u)=a(x)|u|^{p-1}u \) with \( p>1 \)) in unbounded domains. Suitable a priori estimates on \( \Omega \cap B_R \) allow us to obtain a general existence and uniqueness result without growth restrictions at infinity on the solutions. We also obtain estimates on the asymptotic behaviour of solutions when \( |x| \rightarrow +\infty \) and the data \( f \) vanishes outside of some ball \( B_R=\{x:|x|<R\} \).

Problèmes aux limites elliptiques non linéaires dans des domaines non bornés et le comportement asymptotique de ses solutions

Résumé — On utilise des méthodes d’énergie pour étudier les problèmes aux limites elliptiques non linéaires avec un terme surlinéaire (e.g. \( g(x,u)=a(x)|u|^{p-1}u \) avec \( p>1 \)) dans des domaines non bornés. Quelques estimations a priori nous permettent d’obtenir un résultat d’existence et d’unicité général sans conditions de croissance à l’infini sur les solutions. On donne aussi des estimations sur le comportement asymptotique des solutions lorsque \( |x| \rightarrow +\infty \) et la donnée \( f \) s’annule dans \( \mathbb{R}^n-B_R \).

Version française abrégée — On considère le problème aux limites (P) sur un ouvert régulier non borné de \( \mathbb{R}^n \). Dans tout le travail on suppose vérifiée l’hypothèse (1). On introduit les espaces \( V(\Omega)=\{w \in H^1(\Omega): w=0 \text{ sur } \Gamma_D \cap B_R \} \), \( V_{\text{loc}}(\Omega)=\{w: w \in V(\Omega) \text{ pour tout } R>0 \} \) et ses espaces duals \( V^*(\Omega) \) et \( V_{\text{loc}}^*(\Omega) \) respectivement. Pour \( f \in V_{\text{loc}}^*(\Omega) \) on dit que \( u \in V_{\text{loc}}(\Omega) \) est une solution faible de (P) si \( a(x)|u|^{p-1}u \in L_{\text{loc}}^1(\Omega) \) et l’identité (3) est vérifiée pour tout \( \zeta \in V_{\text{loc}}(\Omega) \cap L^\infty(\Omega) \) à support compact. On définit la fonction de localisation \( \Theta_R(x)=\zeta(\frac{|x|}{R}) \) où \( \Theta \in C^0(\mathbb{R}) \) est telle que \( \Theta(0)=1 \) si \( |x| \leq 1/2 \) et \( \Theta(0)=0 \) si \( |x| \geq 1 \).

Notre premier résultat (théorème 1) assure l’existence et l’unicité d’une solution faible u telle que \( a(x)|u|^{p+1} \in L_{\text{loc}}^1(\Omega) \) et qui satisfait (3) pour \( \zeta=\Theta_R \). La démonstration utilise les estimations a priori (6) et (7) pour les solutions des problèmes \( (P_n) \) posés sur \( \Omega_n=\{x \in \Omega: |x|<n\} \). L’hypothèse (1) permet de choisir \( \theta \) tel que \( \theta'(|x|)=O((1-s)^{n-1}) \) et l’important lemme 1 est montré en prenant \( u_n \Theta_n \) comme fonction test. La démonstration de l’existence d’une solution faible de (P) est alors basée sur un procédé d’extraction diagonale appliqué à la suite \( \{u_n\} \). Pour montrer l’unicité on fait \( v=u_1-u_2 \) et en prenant \( \zeta=v \Theta_R \) on voit que les estimations du lemme 1 (avec \( f=0 \)) restent valables pour v ce qui permet de conclure si \( 1<p<(n+2)/(n-2) \) ou \( n=2 \). Si \( p \geq (n+2)/(n-2) \) on suppose, par l’absurde, que \( v(x)>0 \) sur une boule \( B_R(x_0) \). On prend \( \zeta=T_{\text{inf}}(|v-a_+|) \Theta_R(|x|) \) comme fonction test et la contradiction est obtenue avec des estimations similaires à celles du lemme 1 mais cette fois en remplaçant \( p \) par \( q \) avec \( q \in (1, (n+2)/(n-2)) \). Des informations sur quelques résultats précédents dans la littérature sont données dans la remarque 1. En particulier nos résultats généralisent ceux du travail pionnier de Brezis [3].

La deuxième partie de l’article est consacrée à l’étude du comportement des solutions quand \( |x| \rightarrow +\infty \). On fait l’hypothèse (10) et par simplicité on considère seulement le cas \( f=0 \) [équation (11)]. Dans le cas de conditions aux limites de type Dirichlet à l’infini, (12), on donne deux estimations. La première est obtenue par comparaison avec certaines supersolutions (voir théorème 2) et elle est exacte sous des hypothèses convenables (\( \Omega=\mathbb{R}^n \) entre autres). La deuxième, démontrée par une méthode d’énergie, est du type exponentiel (15).

Note présentée par Haïm Brezis.

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grâce à une hypothèse géométrique sur le domaine \( \Omega \) [voir (14)]. Une estimation asymptotique pour le cas du problème de Neumann à l’infini est donnée dans le théorème 4. On montre aussi que les solutions convergent vers zéro quand \( |x| \to +\infty \).

1. Existence, uniqueness and some estimates. — Let \( \Omega \) be a regular open unbounded set of \( \mathbb{R}^n \). We consider the boundary value problem

\[
(P) \quad -Lu + a(x)|u|^{p-1}u = f(x) \quad \text{in } \Omega,
\]

\[u = 0 \quad \text{on } \Gamma_D, \quad \frac{\partial u}{\partial \nu} = 0 \quad \text{on } \Gamma_N,
\]

where \( \partial \Omega = \Gamma_D \cup \Gamma_N \) and \( \Gamma_D \) or \( \Gamma_N \) can be the empty set. The operator \( L \) is given by

\[
Lu = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left( a_{ij}(x) \frac{\partial u}{\partial x_j} \right)
\]

with \( a_{ij} \in L^{\infty}_{loc}(\Omega) \) such that \( \sum_{i,j=1}^n a_{ij}(x) \xi_i \xi_j \geq \lambda |\xi|^2 \), a.e. \( x \in \Omega \), \( \forall \xi \in \mathbb{R}^n \), and for some constant \( \lambda > 0 \). As usual \( \partial u/\partial \nu = \sum_{i,j=1}^n a_{ij}(x) (\partial u/\partial x_j) \gamma_i \) if \( \nu = (\gamma_1, \ldots, \gamma_n) \) is the outward unit normal vector to \( \partial \Omega \). In the following we shall always assume

(1) \( p > 1 \) and \( a \in L^1_{loc}(\Omega) \), \( a(x) \geq a_0 > 0 \) a.e. \( x \in \Omega \) and some constant \( a_0 \).

We introduce the following notation:

\[
B_R = \{ x \in \mathbb{R}^n : |x| < R \}, \quad \Omega_R = \Omega \cap B_R,
\]

\[
V(\Omega_R) = \{ u \in H^1(\Omega_R) : u = 0 \text{ on } \Gamma_D \cap B_R \}
\]

and

\[
V_{loc}(\Omega) = \{ w : w \in V(\Omega_R) \text{ for any } R > 0 \}.
\]

We also consider the dual spaces (for the usual topologies), \( V^*(\Omega_R) \) and \( V^*_{loc}(\Omega) \) respectively. Let

(2) \( f \in V^*_{loc}(\Omega) \).

Definition. — A function \( u \in V_{loc}(\Omega) \) is called a weak solution of problem (P) if

(3) \( a(x)|u|^{p-1}u \in L^1_{loc}(\Omega) \) and

\[
\sum_{i,j=1}^n \int_\Omega a_{ij}(x) \frac{\partial u}{\partial x_j} \frac{\partial \zeta}{\partial x_i} \, dx + \int_\Omega a(x)|u|^{p-1}u \zeta \, dx = \langle f, \zeta \rangle
\]

for any \( \zeta \in V_{loc}(\Omega) \cap L^\infty_{loc}(\Omega) \), \( \zeta \) with compact support.

Let us introduce the cut-off function

(4) \( \Theta_R(x) = 0^2 (|x|/R) \quad \text{for } R > 0 \),

where \( \Theta \in C^\infty(\mathbb{R}) \) is such that \( \Theta(s) = 1 \) if \( |s| \leq 1/2 \) and \( \Theta(s) = 0 \) if \( |s| \geq 1 \).

Theorem 1. — Under the above conditions there exists a unique weak solution \( u(x) \) of problem (P) such that \( a(x)|u|^{p+1} \in L^1_{loc}(\Omega) \) and (3) holds for \( \zeta = u \Theta_R \), for any \( R > 1 \).

To carry out the proof, given \( N > 0 \), we consider the problem

\[
(P_N) \begin{cases}
-Lu_N + a(x)|u_N|^{p-1}u_N = f(x) & \text{in } \Omega_N \\
\frac{\partial u_N}{\partial \nu} = 0 & \text{on } \Gamma_N \cap \partial B_N, \\
u_N = 0 & \text{on } (\Gamma_D \cap \partial B_N) \cup \{ x \in \Omega : |x| = N \}
\end{cases}
\]

\[
|u_N|^{p-1}u_N = f(x) \quad \text{in } \Omega_N,
\]

\[
\frac{\partial u_N}{\partial \nu} = 0 \quad \text{on } \Gamma_N \cap \partial B_N, \quad u_N = 0 \quad \text{on } (\Gamma_D \cap \partial B_N) \cup \{ x \in \Omega : |x| = N \}.
\]
The existence and uniqueness of a solution \( u_N \in H^1(\Omega_N) \) satisfying (P\(_N\)), in the sense that
\[ a(x) \| u_N \|_{L^1(\Omega_N)} \] and the integral identity (3) holds when replacing \( \Omega \) by \( \Omega_N \) for any \( \xi \in H^1(\Omega_N) \cap L^\infty(\Omega_N) \) with \( \xi = 0 \) on \( (\Gamma_D \cap B_N) \cup \{ x \in \Omega : |x| = N \} \), is a consequence of the results of Brezis and Browder [4]. Their results also imply that \( a(x) \| u_N \|_{L^1(\Omega_N)} \) and that the integral identity also holds for \( \xi = u_N \). In fact, if \( 0 < R < N \) we have
\[
\sum_{i,j=1}^n \int_{\Omega_N} a_{ij} \frac{\partial u_N}{\partial x_j} \frac{\partial (u_N \Theta_R)}{\partial x_i} \, dx + \int_{\Gamma_N} a(x) \| u_N \|_{L^1(\Omega_N)} \xi \, dx = \langle f, u_N \Theta_R \rangle.
\]
Indeed, it suffices to take \( \xi = T_m(u_N) \Theta_R \) in the integral identity associated to (3) but on \( \Omega_N \), where \( T_m(x) = \min(m, |x|) \) sign \( r \), if \( r \in \mathbb{R} \). Making \( m \to \infty \) we obtain (5).

We extend the function \( u_N \) by zero over \( \Omega - \Omega_N \) and, for simplicity, we denote again by \( u_N \) this extension. The convergence of \( u_N \) as \( N \to \infty \) will be obtained from

**Lemma 1.** Assume that \( u \in V(\Omega_N), a(x) \| u \|_{L^1(\Omega_N)} \) and the inequality
\[
\sum_{i,j=1}^n \int_{\Omega_N} a_{ij} \frac{\partial u}{\partial x_j} \frac{\partial (u \Theta_R)}{\partial x_i} \, dx + \int_{\Gamma_N} a(x) \| u \|_{L^1(\Omega_N)} \, dx \leq \langle f, u \Theta_R \rangle
\]
holds for some \( f \in V^*(\Omega_N) \) for any \( R \leq N \). Then there exists \( C > 0 \) independent of \( \Omega, R \) and \( u \) such that if \( R \in [1, N] \) we have
\[
\int_{\Omega_N} |\nabla u|^2 \, dx \leq R^{n-2-(q/p-1)} (C + R^{(q-p-1)/2}) \| f \|^2_{V^*(\Omega_N)},
\]
\[
\int_{\Omega_N} |u|^p \, dx \leq R^{-(q-p-1)/2} (C + R^{(q-p-1)/2}) \| f \|^2_{V^*(\Omega_N)}.
\]

**Proof.** Let \( R = 1 \). From (5) and assumption (1) we have
\[
C_1 \int_{\Omega_1} |\nabla u|^2 \theta^2(|x|) \, dx + a_{ij} \int_{\Omega_1} |u|^{p+1} \theta^2(|x|) \, dx \leq 2 \int_{\Omega_1} |\nabla u|^2 \theta^2(|x|) \, dx + \delta \int_{\Omega_1} |u|^{p+1} \theta^2(|x|) \, dx + C_2 \theta^2(|x|) \, dx + C_3 \theta^2(|x|) \, dx
\]
for any \( \delta > 0 \), where we have used the Hölder and the Young inequalities. Choosing \( \theta(\delta) \) such that \( \theta'(\delta) = O((1-x)^m) \), with \( m > 0 \), we have
\[
I = \int_{\Omega_1} |\nabla \theta(|x|)|^{2(p+1)/p} \, dx \leq C_4 \int_{\Omega_1} (1 - |x|)^{2(p+1)/p} \, dx
\]
and thus \( I \) is finite if \( m > (p+1)/p-1 \). The conclusion, for \( R = 1 \), is now obvious. For the general case \( R > 1 \) we introduce the change of variables \( x = R x' \) and \( u(x) = h^{\alpha} \), where \( h = R^{-2(p-1)/p} \), with \( \alpha > 0 \), and thus estimates (6), (7) are obtained from the result for the case \( R = 1 \).

**Proof of the Theorem.** Existence. By lemma 1 we have
\[
\int_{\Omega_{R/2}} (|\nabla u_N|^2 + |u_N|^{p+1}) \, dx \leq C
\]
for some \( C \) depending on \( R \in [1, N] \) but independent of \( N \). By standard results we have that \( \{ u_N \} \) is bounded in \( H^1(\Omega_N) \) and by diagonal extraction is follows that there exists \( u \) and a subsequence such that \( u_N \to u \) in \( H^{1}_{\text{loc}}(\Omega) \) weakly, in \( L^p_{\text{loc}}(\Omega) \) weakly, in \( L^2_{\text{loc}}(\Omega) \).
Using the monotonicity of the function \(F(s) = |s|^{p-1}s\) we arrive at the conclusion.

**Uniqueness.** — Let \(u_1, u_2\) be weak solutions of (P) such that \(a(x)|u_i|^{p+1} \in L^1_{\text{loc}}(\Omega)\) and satisfy (3) for \(\zeta = u_i \Theta_R\) for any \(R > 1\) \((i = 1, 2)\). Let \(v = u_1 - u_2\). Since \(v \in V_{\text{loc}}(\Omega) \cap L^{p+1}_{\text{loc}}(\Omega)\), arguing as in the proof of (5), we can take \(\zeta = v \Theta_R\) in the integral identity (3) associated to \(u_i\). We get

\[
A(x)(y) = \begin{cases} 
0 & \text{if } u_1(x) \neq u_2(x) \\
\int_{\Omega} a(x)|u_1(x) - u_2(x)|^{p+1} d\chi & \text{if } u_1(x) = u_2(x)
\end{cases}
\]

Using \(p > 1\) it is easy to see that \(A(x) \geq A_0\) a.e. on \(\Omega\), for some constant \(A_0 > 0\). Then Lemma 1 can be applied and so estimates (6) and (7) hold by taking \(f = 0\) and replacing \(u\) by \(v\), i.e.

\[
\int_{\Omega} |\nabla v|^2 d\chi \leq C R^{n-2 - (4/p - 1)}, \tag{8}
\]

\[
\int_{\Omega} |v|^{p+1} d\chi \leq C R^{n-2(p+1)/p - 1}. \tag{9}
\]

Assume now \(1 < p < (n+2)/(n-2)\) or \(n = 2\). Then, from (8), making \(R \to \infty\), we conclude that \(\nabla v \equiv 0\) on \(\Omega\) and using (9) we obtain that \(v \equiv 0\). Let \(p \geq (n+2)/(n-2)\) and assume by contradiction that \(v \neq 0\) in \(\Omega\). Without lost of generality we can assume that there exists \(\alpha > 0\) and a ball \(B_r(x_0)\) with \(r > 0\) and \(x_0 \in \Omega\) such that \(v(x) > \alpha\) u.e. \(x \in B_r(x_0)\). It is easy to prove that the function \(w = (v - \alpha)_+\) satisfies the equality

\[
\sum_{i,j=1}^n a_{ij} \frac{\partial w}{\partial x_j} + \int_{\Omega} A(x) |w|^{p} \frac{\partial \Theta_R}{\partial x_j} d\chi = 0
\]

where \(q < (n+2)/(n-2)\). Since \((w + \alpha)^p \leq b > 0\) for any \(w > 0\) and \(q < p\), where \(b\) is a positive constant, from Lemma 1 we have

\[
\int_{\Omega} |w|^{p+1} d\chi \leq C R^{n-2(p+1)/p - 1} \quad \text{as } R \to +\infty.
\]

Therefore \(v \equiv 0\), which contradicts the assumption \(w > 0\) in \(B_r(x_0)\). \(\Box\)

**Remark 1.** — The first result where the existence and uniqueness of a solution of problem (P) (with \(\Gamma_N = \emptyset\)) was given without growth conditions at infinity on \(f\) and solutions is due to Brezis [3]. The main difference with our existence result comes from the fact that we merely use an energy method which allows us to treat a greater class of functions \(f\). On the other hand Brezis' uniqueness result uses an explicit supersolution and his method can not be applied to the case \(\Gamma_N \neq \emptyset\) (other generalizations of the Brezis' result are [1] and [5]).

2. **On the asymptotic behaviour of solutions at infinity.** — We study the behaviour as \(|x| \to +\infty\) of weak solutions of problem (P). As a matter of fact the relevant assumptions will be localized on the set \(\Omega - B_R\), for some \(R > 0\). We assume

\[
u \in L^\infty(\Omega \cap \partial B_R^c).
\]

For simplicity in the statements we shall concentrate our attention in the case \(f \equiv 0\) on \(\Omega - B_R\), i.e. we shall deal with functions \(u \in H^1_{\text{loc}}(\Omega - B_R)\) such that
\( a(x) |u|^\rho \epsilon L^1_{\text{loc}} (\Omega - B_R) \) and satisfy
\[
(11) \quad - Lu + a(x) |u|^{\rho - 1} u = 0 \quad \text{in} \quad \mathcal{D}' (\Omega - B_R).
\]

We start with the case of Dirichlet boundary conditions “near the infinity”, i.e.
\[
(12) \quad u = 0 \quad \text{on} \quad \partial \Omega - B_R.
\]

**Theorem 2.** There exists a constant \( C > 0 \) such that
\[
| u(x) | \leq C |x|^{-2(\rho - 1)} \quad \text{for} \quad a.e. \quad x \in \Omega - B_R.
\]

**Proof.** Let \( N > R' \) and define \( \Omega_N = \{ x \in \Omega : R' < |x| < N \} \). Let \( v_n \) be the weak solution of the problem
\[
(13) \begin{cases}
-L v_n + a(x) |v_n|^{\rho - 1} v_n = 0 & \text{in} \; \Omega_N^c \vspace{1mm} \\
v_n = u & \text{on} \; \Omega \cap \partial B_R, \vspace{1mm} \\
v_n = 0 & \text{on} \; (\partial \Omega \cap (B_R - B_{R'}) \cup (\Omega \cap \partial B_N).
\end{cases}
\]

Consider the function \( V(x) = K |x|^{-2(\rho - 1)} \). It is easy to check (see e.g. Diaz [6], Chap. 1) that \(-LV + a(x) |V|^{\rho - 1} V \geq 0 \) and that \( V(x) \geq \min(x) \) on \( \Omega \cap \partial B_R \) by assuming \( K \) large enough. Then, by the maximum principle we deduce
\[
|v_n(x)| \leq K |x|^{-2(\rho - 1)} \quad a.e. \quad x \in \Omega_N^c.
\]

Letting \( N \to +\infty \) the conclusion holds. \( \blacksquare \)

The result following shows how the “geometry” of \( \Omega \) can have a crucial influence on the decay of \( u \) as \( |x| \to \infty \).

**Theorem 3.** Assume
\[
\text{diameter} (\Omega \cap \{ x : |x| = N \}) \leq T, \quad \forall \; N \geq R'
\]
where the constant \( T \) does not depend on \( N \). Let \( u \) satisfying (10), (11) and (12). Then there exists \( C \) and \( \sigma \), positive constants, such that
\[
|u(x)| \leq C e^{-\alpha |x|} \quad a.e. \quad x \in \Omega - B_R.
\]

**Proof.** Let \( N > R' \) and let \( v_n \) be the solution of \( (P_N^R) \). By (13) we have \( v_n \in L^\infty (\Omega_N^c) \). Taking \( \zeta = v_n e^{\sigma |x|} \) as test function, \( \sigma = \text{const.} > 0 \), we obtain that
\[
C_1 \int_{\Omega_N^c} |\nabla v_n|^2 e^{\sigma |x|} \, dx + a_0 \int_{\Omega_N^c} |\nabla v_n|^{\rho - 1} e^{\sigma |x|} \, dx \leq \frac{\sigma C_2}{2} \int_{\Omega_N^c} |\nabla v_n|^2 e^{\sigma |x|} \, dx + \frac{\alpha}{2} \int_{\Omega_N^c} |v_n|^2 e^{\sigma |x|} \, dx + \int_{\{ x \in \partial \Omega : |x| = R' \}} \frac{\partial v_n}{\partial \nu} v_n e^{\sigma |x|} \, dx.
\]

Using the Friedrich-Poincaré inequality we have
\[
\int_{\Omega_N^c} |v_n|^2 e^{\sigma |x|} \, dx \leq C_4 \int_{\Omega_N^c} |\nabla v_n|^2 e^{\sigma |x|} \, dx
\]
according to assumption (14). Then, since \( \{ v_n \} \) is bounded in \( H^1_{\text{loc}} (\Omega - B_R) \), we obtain
\[
\int_{\Omega_N^c} |\nabla v_n|^2 e^{\sigma |x|} \, dx + \frac{\alpha}{2} \int_{\Omega_N^c} |v_n|^2 e^{\sigma |x|} \, dx \leq C_4
\]
for some \( C_4 > 0 \) independent of \( N \), if we assume \( \sigma \) small enough. Now, let \( x_0 \in \Omega_N^c \).

Since \( v_n \) satisfies the boundary condition \( v_n = 0 \) on \( \Gamma_D \), by the De Giorgi type theorem (see e.g. [8], Theorem 8.17) we have
\[
\sup_{x \in \Omega_N^c} |v_n(x)| \leq C_5 \left[ \int_{\Omega_N^c} |\nabla v_n|^{\rho + 1} \, dx \right]^{(\rho + 1)} \leq C_6 \left[ e^{\alpha |x_0| - 1} \int_{\Omega_N^c} |v_n|^{\rho + 1} e^{\sigma |x|} \, dx \right]^{(\rho + 1)} \leq C_7 \exp \left\{ \frac{-\alpha}{(p + 1)} |x_0| \right\}
\]
Making $N \to \infty$ we get the conclusion. □

Concerning the case of Neumann boundary conditions “near the infinity”

$$\frac{\partial u}{\partial v} = 0 \quad \text{on} \quad \partial \Omega - B_R.$$  \hspace{1cm} (16)

similar arguments leads to the following result

**Theorem 4.** Let $u$ satisfying (10), (11) and (16). Then

$$|u(x)| \leq C|x|^{1-\beta}, \quad \text{a.e.} \quad x \in \Omega - B_R,$$

where $\beta = [p(n-2)-(n+2)]/\beta^2 - 1$. Moreover, if $u \in C^{2,\beta}(\Omega - B_R)$ for some $\mu \in (0,1)$ then $|u(x)| \to 0$ as $|x| \to \infty$.

The details of the proof can be found in [7].

**Remark 2.** Section 2 generalizes in different ways several results by Kondratiev and Oleinik [9] related to the case of unbounded cylindrical domains $\Omega = \omega \times \mathbb{R}$ or $\Omega = \omega \times \mathbb{R}$ with $\omega$ an open bounded set of $\mathbb{R}^{N-1}$. When $L = \Delta$ and $\Omega$ is a cylinder the asymptotic behaviour of solutions have been studied from the point of view of dynamical systems since many years ago. Some results in this direction can be found in the article of Brada [2] where this author develops ideas already introduced in Veron [10], [11].

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