HYPERBOLIC CONSERVATION LAWS

$$\partial_t U(x,t) + \sum_{\alpha=1}^{m} \partial_\alpha G_\alpha(U(x,t)) = 0$$

$x \in \mathbb{R}^m$, $t \in \mathbb{R}$, $\partial_t = \dfrac{\partial}{\partial t}$, $\partial_\alpha = \dfrac{\partial}{\partial x_\alpha}$, $\alpha = 1, \dots, m$

$U \in \mathbb{R}^n$, $\quad G_\alpha : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, $\alpha = 1, \dots, m$

Hyperbolic:

For all $U \in \mathbb{R}^n$ and $\nu \in S^{m-1}$, the matrix

$$\Lambda(U;\nu) = \sum_{\alpha=1}^{m} \nu_\alpha \, DG_\alpha(U)$$

has real eigenvalues and n linearly independent eigenvectors

ISENTROPIC GAS DYNAMICS

$$\begin{cases} \partial_t \rho + \operatorname{div} m = 0 \\ \partial_t m + \operatorname{div}\left(\frac{1}{\rho} m \otimes m\right) + \operatorname{grad} p(\rho) = 0 \end{cases}$$

$$p(\rho) = \rho^2 \varepsilon'(\rho)$$

ρ : density

m : momentum

p : pressure

ε : internal energy

Hyperbolic $\Leftrightarrow$ $p'(\rho) > 0$

ELECTRODYNAMICS FOR BORN-INFELD MEDIA

$$\begin{cases} \partial_t B = -\operatorname{curl} E \\ \partial_t D = \operatorname{curl} H \end{cases}$$

$$Q = D \times B$$

$$\eta = \left[1 + |B|^2 + |D|^2 + |Q|^2 \right]^{\frac{1}{2}}$$

$$E = \frac{\partial \eta}{\partial D} = \frac{1}{\eta} \left[D + B \times Q \right]$$

$$H = \frac{\partial \eta}{\partial B} = \frac{1}{\eta} \left[B - D \times Q \right]$$

E : electric field (3-vector)

H : magnetic field (3-vector)

B : magnetic induction (3-vector)

D : electric displacement (3-vector)

Q : Poynting vector (3-vector)