

Workshop

Operators and Banach lattices II



Facultad de CC. Matemáticas
Universidad Complutense de Madrid
17-18 November 2016



Speakers:

Antonio Avilés (Universidad de Murcia)
Óscar Blasco (Universidad de Valencia)
Christina Brech (Universidade de São Paulo)
Jesús Castillo (Universidad de Extremadura)
Guillermo Curbera (Universidad de Sevilla)
Manuel González (Universidad de Cantabria)
Jordi López-Abad (Université Paris VII)
Ginés López Pérez (Universidad de Granada)
Fernando Lledó (Universidad Carlos III de Madrid)
Martín Mathieu (Queen's University Belfast)
Yves Raynaud (CNRS/Université Paris VI)
Javier Soria (Universidad de Barcelona)
Onno van Gaans (Universiteit Leiden)

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WORKSHOP PROGRAMME
 Room Miguel de Guzmán
 Facultad de Ciencias Matemáticas
 Universidad Complutense de Madrid

SCHEDULE	Thursday 17 November	
9:50	Welcome and registration	
10:00-10:45	Y. RAYNAUD	On certain asymptotically hilbertian modular spaces and their elementary classes
10:45-11:30	J. SORIA	Mixed norms and iterated rearrangements
11:30-12:00	COFFEE BREAK	
12:00-12:45	G. CURBERA	A classical result of Hardy motivated the study of the Cesàro operator
12:45-13:30	O. BLASCO	Summing multinorms defined by Orlicz spaces and symmetric sequence spaces
13:30-15:00	LUNCH	
15:00-15:45	G. LÓPEZ	Looking for a Lipschitz characterization of some well known properties in Banach spaces
15:45-16:30	C. BRECH	Decomposing l_∞/c_0 into infinitely many copies of a fixed space
16:30-17:00	COFFEE BREAK	
17:00-17:45	J. CASTILLO	Differentials of complex interpolation processes
20:30	CONFERENCE DINNER	

SCHEDULE	Friday 18 November	
10:00-10:45	O. VAN GAANS	Positive operators on pre-Riesz spaces
10:45-11:30	J. LÓPEZ-ABAD	Approximate Ramsey properties of operator spaces and systems
11:30-12:00	COFFEE BREAK	
12:00-12:45	F. LLEDÓ	Operators, Følner sequences and paradoxicality
12:45-13:30	A. AVILÉS	Extension of operators in Banach spaces that contain (or not) the space of bounded sequences
13:30-15:00	LUNCH	
15:00-15:45	M. GONZALEZ	Convolution operators on $L_1(G)$ which are tauberian or cotauberian
15:45-16:30	M. MATHIEU	Elementary operators - still not elementary?
16:30-17:00	Open problem session	
17:00	WINE	

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Abstracts

On certain asymptotically hilbertian modular spaces and their elementary classes

Yves Raynaud

(Institut de Mathématiques de Jussieu)

Two Banach spaces X, Y are said to be *elementarily equivalent* if for some ultrafilter \mathcal{U} their respective ultrapowers $X_{\mathcal{U}}$ and $Y_{\mathcal{U}}$ are linearly isometric. The elementary class of X consists of all Banach spaces that are elementarily equivalent to X . The aim of the work is to provide non trivial examples of separable Banach spaces the elementary class of which contains exactly one element of density character κ for each uncountable cardinal κ (a trivial example is the Hilbert space). One kind of these examples consists simply of certain Nakano spaces of sequences. The general pattern of our examples is a vectorial version of the preceding ones. Joint work with C. W. Henson, (Urbana-Champaign, USA)

Mixed norms and iterated rearrangements

Javier Soria

(Universidad de Barcelona)

It was proved in [1] that mixed norm spaces $X[Y]$ are never rearrangement invariant, except when $X = Y = L^p$. This fact motivates the study of the behavior of iterated rearrangements on $X[Y]$. We will show that, for the mixed norm Lebesgue spaces, the only example of iterated invariance (off the diagonal) occurs on $L^p[L^\infty]$. For the remaining cases, we prove sharp estimates, and find the optimal range of indices, for the comparison of the mixed norm of a function and its iterated rearrangement. This is a joint work with Viktor Kolyada.

References:

1. A. Boccuto, A. V. Bukhvalov, and A. R. Sambucini, *Some inequalities in classical spaces with mixed norms*, Positivity **6** (2002), no. 4, 393–411.
 2. V. Kolyada and J. Soria, *Mixed norms and iterated rearrangements*, Z. Anal. Anwend. **35** (2016), 119–138.
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A classical result of Hardy motivated the study of the Cesàro operator

Guillermo Curbera
(Universidad de Sevilla)

A classical result of Hardy motivated the study of the Cesàro operator

$$\mathcal{C} : f \mapsto \mathcal{C}(f)(x) := \frac{1}{x} \int_0^x f(t) dt,$$

in the L^p spaces, leading to the spaces $Ces_p := \{f : \mathcal{C}(|f|) \in L^p\}$. These spaces have been extensively studied by Astashkin, Kaminska, Lesnik, Maligranda and others.

We change the focus and look at spaces obtained by considering operators T other than the Cesàro operator and function spaces X other than L^p , resulting in the spaces

$$[T, X] := \{f : T(|f|) \in X\}.$$

In this talk we consider the weak Banach-Saks and the Radon-Nikodym properties for the spaces $[T, X]$.

Joint work with Werner J. Ricker from the Katholische Universität Eichstätt (Germany).

Summing multinorms defined by Orlicz spaces and symmetric sequence spaces

Óscar Blasco
(Universidad de Valencia)

We develop the notion of (X_1, X_2) -summing power-norm based on a Banach space E where X_1 and X_2 are symmetric sequence spaces. We study the particular case where X_1 and X_2 are Orlicz spaces ℓ_Φ and ℓ_Ψ respectively and analyze when the (Φ, Ψ) -summing power-norm becomes a multinorm. In the case that E is also a symmetric sequence space L we compute the precise value of $\|(\delta_1, \dots, \delta_n)\|_n^{(X_1, X_2)}$ where (δ_k) stands for the canonical basis of L , extending the known results for the (p, q) -summing power-norm based on ℓ_r which now corresponds to $X_1 = \ell_p$ and $X_2 = \ell_q$ and $E = \ell_r$.

Looking for a Lipschitz characterization of some well known properties in Banach spaces

Ginés López

(Universidad de Granada)

We study the topology in the unit sphere of a Banach space generated by the family of Lipschitz slices, as a subbasis for this topology. We consider the relation between the above topology and the weak topology and as a consequence, we get natural characterizations in terms of Lipschitz slices for some well known properties in Banach spaces, for example Radon Nikodym and Daugavet properties. This shows that the above properties only depend on the uniformity of the Banach space, given by the linear and metric structures. Motivated by some problems about embeddings of some metric spaces in Banach spaces and the existence of a metric characterization of the Radon-Nikodym property, we also propose some natural question about the above topic.

Decomposing ℓ_∞/c_0 into infinitely many copies of a fixed space

Christina Brech

(Universidade de São Paulo)

We will present the independence result concerning the possibility of decomposing ℓ_∞/c_0 isomorphically as an ℓ_∞ -sum of infinitely many copies of a fixed Banach space. While an old result of Drewnowski and Roberts guarantees that under the continuum hypothesis the space ℓ_∞/c_0 is isomorphic to the ℓ_∞ -sum of itself, together with Piotr Koszmider we proved the consistency of this not being possible. This is done by constructing a model where the ℓ_∞ -sum of $c_0(\mathfrak{c})$ cannot be embedded into ℓ_∞/c_0 .

Differentials of complex interpolation processes

Jesús M. F. Castillo

(Universidad de Extremadura)

The purpose of the talk is to present recent advances in the understanding of differentials of complex interpolation processes, as initiated by Kalton, and Rochberg and Weiss. Recall that a twisted sum of a Banach space X is a Banach space Ω_X containing a subspace isomorphic to X so that Ω_X/X is isomorphic to X . For our purposes, the simplest example is the Kalton-Peck [6] space Z_2 that is a twisted Hilbert space (i.e., a twisted sum of Hilbert spaces ℓ_2).

On the other hand, it is well known that given a complex interpolation scale, a kind of derivation process produces twisted sums. To be more precise, given an interpolation couple (X_0, X_1) for which complex interpolation produces spaces X_θ for $0 < \theta < 1$, the derivation process at θ produces twisted sums Ω_θ of X_θ . In this context, the space Ω_θ is called the derived space (of the scale) at θ . Again, the simplest example is the space Z_2 , which appears as the derived space at $1/2$ of the scale of ℓ_p -spaces when starting with the couple (ℓ_1, ℓ_∞) .

Kalton [4, 5] started the study of differential processes and did a tremendously deep work in the context of Köthe function spaces. We will consider recent advances regarding the structure and stability of differential processes both in the context of Köthe spaces and in full generality.

This is a joint work with Valentin Ferenczi and Manuel González.

References:

1. F. Cabello, J.M.F. Castillo, N. J. Kalton, *Complex interpolation and twisted twisted Hilbert spaces*. Pacific J. Math. (2015) 276 (2015) 287 - 307.
2. J. M. F. Castillo, *Simple twist of K* , in Kalton Selecta vol.2., pp. 251-254, Contemporary Mathematicians, Birkhauser (2016).
3. J.M.F. Castillo, V. Ferenczi, M. Gonzalez, *Singular exact sequences generated by complex interpolation*, Trans. Amer. Math. Soc. (2016) (in press)
4. N. J. Kalton, *Nonlinear commutators in interpolation theory*, Mem. Amer. Math. Soc., 385 (1988)
5. N.J. Kalton, *Differentials of complex interpolation processes for Köthe function spaces*, Trans. Amer. Math. Soc. 333 (1992) 479–529.
6. N.J. Kalton and N.T. Peck, *Twisted sums of sequence spaces and the three space problem*, Trans. Amer. Math. Soc. 255 (1979) 1–30.
7. R. Rochberg and G. Weiss, *Derivatives of analytic families of Banach spaces*, Ann. of Math. 118 (1983) 315–347.

Positive operators on pre-Riesz spaces

Onno van Gaans
(Universiteit Leiden)

The theory of positive operators on vector lattices and Banach lattices is very rich. Many of the properties that are studied do also make sense on more general partially ordered vector spaces. For instance, there is a natural notion of disjointness for elements of the space of continuously differentiable functions on an interval and thus one could study disjointness preserving operators on that space. It turns out that notions such as disjointness, band, ideal, and lattice homomorphism have generalizations with suitable properties to partially ordered vector spaces that are order dense subspaces of vector lattices. Such spaces are called pre-Riesz spaces. Results as in the lattice case for operators on pre-Riesz spaces are much more difficult to obtain. We will consider some of the difficulties and we will also consider some initial results for special cases that suggest that there might be a rich theory also in the setting of pre-Riesz spaces.

Approximate Ramsey properties of operator spaces and systems

Jordi López-Abad
(Université Paris Diderot)

We compute the universal minimal flow of the automorphism groups of certain operator spaces and systems. The commutative examples are the Gurarij space and the Poulsen simplex and, in general, we study their non-commutative analogues. The Gurarij space \mathbb{G} is the unique separable approximately ultrahomogeneous Banach space that contains ℓ_n^∞ for every $n \in \mathbb{N}$, while \mathbb{P} is the unique metrizable Choquet simplex with dense extreme boundary. The group $\text{Aut}(\mathbb{G})$ of surjective linear isometries of \mathbb{G} is shown to be extremely amenable, by proving the approximate Ramsey property of the class of finite-dimensional Banach spaces. Similarly the stabilizer $\text{Aut}_p(\mathbb{P})$ of an extreme point p of \mathbb{P} is proven to be extremely amenable, by establishing the approximate Ramsey property of the class of Choquet simplices with a distinguished point. It is then deduced that the universal minimal flow of $\text{Aut}(\mathbb{P})$ is \mathbb{P} itself. More generally, we prove that for any closed face F of \mathbb{P} , the pointwise stabilizer $\text{Aut}_F(\mathbb{P})$ is extremely amenable. We will also discuss the approximate ultrahomogeneity of the spaces $L_p[0, 1]$ and the topological dynamics of their groups of isometries.

Operators, Følner sequences and paradoxicality

Fernando Lledó
(Universidad Carlos III de Madrid)

In this talk we will introduce the notion and first properties of Følner sequences in the context of Operator Theory and Operator Algebras. Let $\mathcal{T} \subset \mathcal{B}(\mathcal{H})$ be a set of bounded linear operators acting on a complex separable Hilbert space \mathcal{H} . A sequence of non-zero finite rank orthogonal projections $\{P_n\}_{n \in \mathbb{N}}$ is called a Følner sequence for \mathcal{T} , if

$$\lim_n \frac{\|TP_n - P_nT\|_2}{\|P_n\|_2} = 0, \quad T \in \mathcal{T},$$

where $\|\cdot\|_2$ is the Hilbert-Schmidt norm. Følner sequences generalize the notion of quasi-diagonality for operators and can also be applied to spectral approximation problems. We will relate the notion of Følner sequence for operators to the existence of amenable traces and to the class of finite operators in the sense of Williams. Finally, we will also mention in some examples the relation of Følner sequences with properly infinite C^* -algebras.

References:

1. P. Ara, K. Li, F. Lledó and J. Wu, *Amenability of coarse spaces and K -algebras*, preprint:math.RA/1607.00328
2. P. Ara and F. Lledó, *Amenable traces and Følner C^* -algebras*, *Expo. Math.* **32** (2014) 161-177.
3. F. Lledó and D. Yakubovich, *Følner sequences and finite operators*, *J. Math. Anal. Appl.*, **403** (2013) 464-476.

4. N.P. Brown, *Invariant means and finite representation theory of C^* -algebras*, Mem. Am. Math. Soc. **184** (2006) no. 865, 1-105.
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Extension of operators in Banach spaces that contain (or not) the space of bounded sequences

Antonio Avilés
(Universidad de Murcia)

We shall review some properties of extension of operators in Banach spaces that reflect general categorial concepts. As we shall see, some of the problems that arise in this context are sensitive to different axioms of set theory.

Convolution operators on $L_1(G)$ which are tauberian or cotauberian

Manuel González
(Universidad de Cantabria)

We study the convolution operators T_μ acting on the convolution algebra $L_1(G)$, where G is a locally compact abelian group and μ is a complex Borel measure on G .

We show that the operator T_μ is Fredholm when it is cotauberian; i.e. the conjugate operator T_μ^* acting on $L_\infty(G)$ is tauberian. Moreover, assuming that T_μ is tauberian, we show that T_μ is invertible when G is non-compact, that T_μ is Fredholm when it has closed range and G is compact, and in the remaining case when G is compact and $R(T_\mu)$ is not closed, we prove that T_μ is Fredholm when the singular continuous part of μ with respect to the Haar measure on G is zero.

Joint work with Liliana Cely and Elói M. Galego (I.M.E. São Paulo, Brazil).

Elementary operators—still not elementary?

Martin Mathieu
(Queen's University Belfast/MRIA)

Many properties of elementary operators on Banach algebras have been studied extensively by a large number of authors over several decades but still they defy a full structural understanding maybe the reason being that they appear naturally in many guises in many situations. I shall report on recent work with Nadia Boudi (2015) and my latest PhD student Matthew Young (2016) in which we try to understand when elementary operators are spectrally bounded or spectrally isometric. As so often, definitive results are only achieved for elementary operators of small length.

List of participants

- Abderramn Amr Rey (Universidad Complutense de Madrid).
- Vicente Asensio López (Universidad Politécnica de Valencia).
- Antonio Avilés (Universidad de Murcia).
- Joan Carles Bastons (Universidad Autónoma de Madrid).
- Glenier L. Bello (ICMAT - Universidad Autónoma de Madrid).
- Óscar Blasco (Universidad de Valencia).
- Christina Brech (Universidade de São Paulo, Brazil).
- Félix Cabello (Universidad de Extremadura).
- Jesús M. F. Castillo (Universidad de Extremadura).
- Guillermo Curbera (Universidad de Sevilla).
- Andoni De Arriba De La Hera (Universidad Complutense de Madrid).
- Julio Flores (Universidad Rey Juan Carlos).
- Manuel González (Universidad de Cantabria).
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- Diego Yáñez (Universidad de Extremadura).

