Negligible influence of small enough jitter in the neuron geometry on the extracellular potential

Reviewer 3: supplemental material for the manuscript by Makarova et al., 2011, “Parallel readout of the pathway-specific inputs to laminated brain structures”

A. Initial definitions

Let

$$\varphi^0(r,t) = -\frac{1}{4\pi\sigma}\sum_{n=1}^{N}\sum_{c=1}^{C}\frac{I_{nc}^0(t)}{d_{nc}^0} \tag{1}$$

be the potential created by an aggregate of N identical, perfectly aligned neurons with C compartments (see also Eq. (3) in the manuscript). Under the perfect alignment we understand parallel alignment of the neurons over (vertical) z-axis with no rotation in (x,y)-plane. Somas of all neurons have the same z position but can be randomly distributed in (x,y)-plane. In Eq. (1) $d_{nc}^0 = \|r_{nc}^0 - r\|$ is the Euclidean distance between the compartment nc and the recording point, $I_{nc}^0(t)$ is the transmembrane current generated by the compartment, and σ is the (constant) conductivity of the extracellular space. We notice that

$$d_{nc}^0 \geq \delta > 0, \ \ \forall n, c \tag{2}$$

where δ is a small enough number, otherwise the model of the potential (1) is not valid. Under the assumption of the same synaptic input Eq. (1) can be simplified

$$\varphi^0(r,t) = -\frac{1}{4\pi\sigma}\sum_{c=1}^{C} I_c^0(t) R_c^0 \tag{3}$$

where $R_c^0 = \sum_{n=1}^{N}\frac{1}{d_{nc}^0}$ is the cumulative inverse distance to the c-th compartment of all neurons (this sum converges to a constant depended on the mean intersoma distance).

B. Jitter in the compartment’s diameter

Let us consider the case of variation of the diameter of the compartments among different neurons in the aggregate. The diameter of the c-th compartment of the n-th neuron is given by:

$$a_{nc} = a_{nc}^0 + \epsilon\xi_{nc} \tag{4}$$

where a_{nc}^0 is the diameter of the original (unperturbed) compartment, $0 < \epsilon$ is the smallness parameter and ξ_{nc} is a random zero mean variable describing the perturbation.

The compartmental transmembrane current is proportional to the compartment’s diameter: $I_{nc} \propto a_{nc}$. Thus the variation (4) induces variation in the compartmental currents of the order of magnitude ϵ (to obtained exact values, we should also take into account the readjustment of the currents due to their intracellular interaction). Using the Taylor expansion we can write:

$$I_{nc}(t) = I_{nc}^0(t) + \epsilon J_{nc}(t) + O(\epsilon^2) \tag{5}$$

where $J_{nc}(t)$ are the transmembrane currents induced by the variation of the diameter of the compartments. Then the new currents given by (5) lead to changes in the potential:

$$\varphi(r,t) = \varphi^0(r,t) + \epsilon\psi(r,t) + O(\epsilon^2) \tag{6}$$

where

$$\psi(r,t) = -\frac{1}{4\pi\sigma}\sum_{n=1}^{N}\sum_{c=1}^{C}\frac{J_{nc}(t)}{d_{nc}^0} \tag{7}$$

is the first order correction to the potential. In Eq. (6) we notice that the correction of the potential is of the order ϵ. Besides, since J_{nc} is a random variable of its indexes the sum (7) will be significantly smaller in magnitude than the sum (1). To demonstrate this statement we introduce new random variables:

$$\eta_c = \sum_{n=1}^{N} \frac{J_{nc}}{d_{nc}^0} \quad (8)$$

Since $N \gg 1$ ($N = 16966$ in simulations), the central limit theorem is applied and we have

$$\eta_c \sim \mathcal{N}(\mu_\eta, \lambda_\eta) \quad (9)$$

One can easily show that the mean value of η_c is zero ($\mu_\eta = 0$), whereas the standard deviation is given by

$$\lambda_\eta = \lambda_J \sqrt{\sum_{n=1}^{N} \frac{1}{\left(d_{nc}^0\right)^2}} \quad (10)$$

where λ_J is the standard deviation of the random correcting transmembrane currents J_{nc}. Assuming that λ_J does not depend on the compartment's number (which is true at least at the first order of magnitude) we then obtain the final expression for the correcting potential:

$$\psi \sim \mathcal{N}\left(0, \lambda_\psi\right), \quad \lambda_\psi = \lambda_J \sqrt{C \sum_{n=1}^{N} \frac{1}{\left(d_{nc}^0\right)^2}} \quad (10)$$

Thus small enough jitter in the compartment' diameters makes no significant change in the extracellular potential.