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Prediction-for-CompAction: navigation in social environments using generalized cognitive maps

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Abstract The ultimate navigation efficiency of mobile robots in human environments will depend on how we will appraise them: merely as impersonal machines or as human-like agents. In the latter case, an agent may take advantage of the cooperative collision avoidance, given that it possesses recursive cognition, i.e., the agent's decisions depend on the decisions made by humans that in turn depend on the agent's decisions. To deal with this high-level cognitive skill, we propose a neural network architecture implementing Prediction-for-CompAction paradigm. The network predicts possible human–agent collisions and compacts the time dimension by projecting a given dynamic situation into a static map. Thereby emerging compact cognitive map can be readily used as a "dynamic GPS" for planning actions or mental evaluation of the convenience of cooperation in a given context. We provide numerical evidence that cooperation yields additional room for more efficient navigation in cluttered pedestrian flows, and the agent can choose path to the target significantly shorter than a robot treated by humans as a functional machine. Moreover, the navigation safety, i.e., the chances to avoid accidental collisions, increases under cooperation. Remarkably, these benefits yield no additional load to the mean society effort. Thus, the proposed strategy is socially compliant, and the humanoid agent can behave as "one of us."

Keywords Navigation in dynamic situations · Cognition · Compact cognitive maps · Dynamic GPS · Decision making · Nonlinear dynamics

1 Introduction

Flexible and efficient navigation in dynamically changing situations is a key step toward developing artificial agents capable of sharing with people the same social environment. While the electromechanical platforms of modern robots advance quite rapidly, their cognitive abilities remain relatively limited. This restricts significantly the robot deployment in our daily life. There is a certain lack of mathematical models simulating the human–robot cooperation when the robot moves through a space structured by the activity of humans (Trautman et al. 2013). Existing approaches can be roughly subdivided into three main classes: (i) heuristic deterministic algorithms, (ii) probabilistic models, and (iii) neural networks mimicking cognitive processes in living beings. The latter approach is relatively new and less explored. However, growing neurophysiological data encourages modeling natural cognition from the first biophysical principles (Schmidt and Redish 2013).

The cooperation among human pedestrians has been initially addressed by means of heuristic arguments. Deterministic models simulating interaction among humans in the framework of self-driven many-particle systems reproduced some phenomena of self-organization and disasters observed in human crowds (Helbing and Molnar 1995; Helbing et al. 2000; Dyer et al. 2008). In particular, Moussaid et al. (2011) recently proposed a simple multi-agent

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model based on two empirical observations: In normal conditions, humans try to maintain comfortable velocity and follow straightforward directions to the target but keeping prudent distance with obstacles. The model reproduces collective collision avoidance, formation of unidirectional lanes, and stop-and-go waves. However, the local decision making behind such algorithms may frequently lead to suboptimal behaviors and unnecessary random maneuvers in situations without feasible solutions (Conn and Kam 1998). Other models exploit the idea of creating potential fields on the basis of proximity among pedestrians and rapid replanning. They provide suitable trajectories in low-density crowds (Svenstrup et al. 2010), but if the environment surpasses a certain complexity level, there appears the “freezing” robot problem when all forward paths are considered unsafe (Trautman and Krause 2010). Then, a probabilistic model simulating the joint collective avoidance can “make” room for safe trajectories (Trautman et al. 2013). For a similar purpose, Kuderer et al. (2012) used empirical human’s trajectories for fitting a model of the probability distribution that underlies human navigation behavior.

Heuristic algorithms and probabilistic models enlightened the pivotal role of cooperation, but they did not tackle the problem of how cognition appears. A neural network approach simulating cognition and cooperation as an emerging property (i.e., not rule-imposed) has been significantly less explored. Existing neural network models are mostly devoted to imitation of specific human actions and comprehension of human intentions (Dillmann et al. 2004). Architectures relying on a bottom-up approach to robot navigation still show limited cognitive abilities (Schilling et al. 2013). Besides, existing neural networks mainly deal with noncooperative navigation (Villacorta-Atienza et al. 2010; von Hundelshausen et al. 2011).

In this work, we study how a neural network generating so-called *compact cognitive maps* supports cognitive processes that enable global decision making for navigation in social environments. In order to dissect the problem into basic building blocks, we consider two opposite scenarios: coexistence of robot with humans (no collaboration from humans is expected) and cooperation (both the robot and humans change trajectories to avoid collisions). In the latter case, the problem is equivalent to the cooperation of an agent possessing Theory of Mind or ToM (Premack and Woodruff 1978) that considers other agents as also having ToM. Then, *recursive cognition* emerges: an action of an agent will depend on decisions made by other agents that in turn depend on the decisions made by the agent and so on (Dennett 1987). Therefore, a neural network exhibiting recursive cognition should encapsulate the dynamics of decision-making processes of similar or higher complexity.

Here, we show that compact cognitive maps support recursive cognition, which leads to significantly different perception and behaviors both in structured and unstructured crowds. We conclude that, in general, cooperation offers significant benefits to the robot. Remarkably, these benefits assume no additional load to the society effort, i.e., under cooperative navigation, the agent does not destabilize the society and can behave as “one of us.”

1.1 Prediction-for-CompAction paradigm (PfCA)

In the last decades, diverse experimental findings provided insight into the neural mechanisms of cognition. It has been shown that for navigation in space animals use a GPS-like abstract representations of the environment, called cognitive maps (O’Keefe and Nadel 1978; Schmidt and Redish 2013). A cognitive map contains critical information for planning movements in space, such as the subject location (codified by place cells), objects obstructing free ways (boundary cells), and space metric (grid cells) (Pfeiffer and Foster 2013). This concept has been successfully used for robot navigation, but mostly in static environments (Franz and Mallot 2000; Meyer and Filliat 2003).

Growing experimental evidence suggests that the neural structures generating cognitive maps (e.g., the hippocampus and medial prefrontal cortex) also participate in the representation of dynamic situations. For instance, place cells also encode speed, turning angle, and direction of moving objects (Ho et al. 2008). At larger scale, chemical damage of the rat hippocampus impairs avoidance of moving obstacles, with no effect on other critical abilities (Telensky et al. 2011). Thus, in mammals, cognition of dynamic situations is built over cognitive maps and involves coordinated network activity of entire brain areas.

In order to generalize the cognitive maps to *dynamic* environments, recently we have proposed an approach, called Prediction-for-CompAction (PfCA) (Villacorta-Atienza et al. 2010). The PfCA postulates that our brain does not explicitly code the time dimension of dynamic situations. Instead, it forecasts locations where collisions with real obstacles may occur and then transforms them into *effective* obstacles. We note that in static environments, effective obstacles coincide with real ones and the compact cognitive maps are naturally reduced to the ordinary cognitive maps (Villacorta-Atienza et al. 2010). Therefore, the compact cognitive maps extend the properties of ordinary cognitive maps to dynamic situations. They act as a dynamic GPS, providing us with the information required for navigation in time-changing situations.

1.2 Two complementary concepts of navigation in social environments

As we mentioned above, there exist two navigational scenarios: coexistence and cooperation. As we will show below, they are complementary and the choice between them depends on the context.

1.2.1 Coexistence: machine avoids us (AvUs)

A robot regarded by humans merely as a functional machine should not expect any cooperation from people during its ordinary tasks. The machine must move among humans without disturbing us. We will call such a noncooperative behavioral paradigm a *machine Avoids Us* (AvUs). Most of the contemporary robot navigation strategies implicitly implement AvUs, assuming that the robot must "do all the work" to navigate safely, avoiding collisions (Philippsen and Siegwart 2003; Ziebart et al. 2009; Villacorta-Atienza et al. 2010).

Based on the PfCA concept, in our previous works, we have proposed a neural network for cognitive navigation in noncooperative but dynamic situations (Villacorta-Atienza et al. 2010; Makarov and Villacorta-Atienza 2011). We assumed that the time evolution of the environment cannot be altered by the agent's behavior. Thus, any recursive interaction between the agent and its environment, e.g., cooperation or competition, has been left outside the model. In Sect. 2, we will briefly revisit our previous works and discuss key points of the PfCA concept and compact cognitive maps.

1.2.2 Cooperation: machine cooperate with us (CoUs)

The AvUs strategy is sensible for robots with limited embodied cognition. However, this paradigm contrasts with real human behavior. People are predisposed to ascribe a mind to artifacts exhibiting a certain degree of social connections and human likeness (Waytz et al. 2010). An artificial agent looking, moving, and behaving as a human may elicit cooperation, so the robot-society loop closes. At the risk of a collision, a human pedestrian will cooperate with a humanoid agent and deviate from his initial trajectory. We will call this paradigm a *machine Cooperates with Us* (CoUs).

A remarkable difference of CoUs from AvUs strategy is that the former requires recursive higher-level cognition. Earlier a recursive ToM has been used for algorithmic modeling of collision avoidance (Takano and Arita 2006). In particular, it has been shown that the algorithm performance depends on the level of recursion. In this work, for the first time, we introduce the human–robot cooperation in the concept of compact cognitive maps and provide a neural network basis for ToM recursion (Sect. 3). Then, using numerical simulations, we show that the network supports robot–human recursive cognition (Sect. 4). Finally, in Sect. 5, we discuss the results.

2 Cognition through compact cognitive maps

Neural network implementation of cognition of static and dynamic situations may differ significantly. In this context, the concept of compact internal representation provides an elegant way to unify the description of both static environments and dynamic situations (Villacorta-Atienza et al. 2010).

2.1 Cognitive maps for static environments

Let us consider a situation sketched in Fig. 1a. A walking humanoid agent comes across two obstacles: a human and a chair. Both obstacles are immobile, and therefore, the agent is in a *static environment*. Then, the navigation can be fulfilled by using standard cognitive maps [see e.g., (Franz and Mallot 2000)]. Earlier we provided a neural network implementation of this general concept (Villacorta-Atienza et al. 2010).

In real space, the chair occupies some space, whereas the agent and the human are represented by their personal areas (Hall 1963) (Fig. 1b, left panel). To represent "mentally" the real space, we now introduce a 2D neural network, an $(n \times n)$-lattice of locally coupled neurons (Fig. 1b, right panel). This, so-called causal neural network (CNN, "Appendix A"), receives as an input the spatial configuration of the real space. In the network space

$$D = \{(i, j) : \quad i, j = 1, 2, \dots n\} \tag{1}$$

the agent is reduced to a single neuron, while its dimension is properly added to the obstacles' dimensions, thus proportionally increasing their sizes (Lozano-Perez and Wesley 1979).

In the network space D, the agent has to create a cognitive map of the environment. It is fulfilled by virtual simulation of all possible agent's movements using a wave process. Since the agent can walk in any direction, its virtual positions (locations occupied by virtual agents) at the next time step will form a circle with the agent in the center (Fig. 1c). The radius of this circle will grow with time as virtual agents will move away from the center. Thus, the process of mental exploration of the environment can be described by a solitary wave propagating in the network outward the agent's initial position. The wavefront detects all obstacles, rounds them, and hence finds possible paths among them. We note that this procedure is computationally/biologically efficient, since the mental exploration is achieved in a single run independently on the complexity of the environment.

Each neuron (i, j) records the time instants c_{ij} when the wavefront passes through it. Thus, we create a 2D potential profile

$$c : D \to \mathbb{R} \tag{2}$$

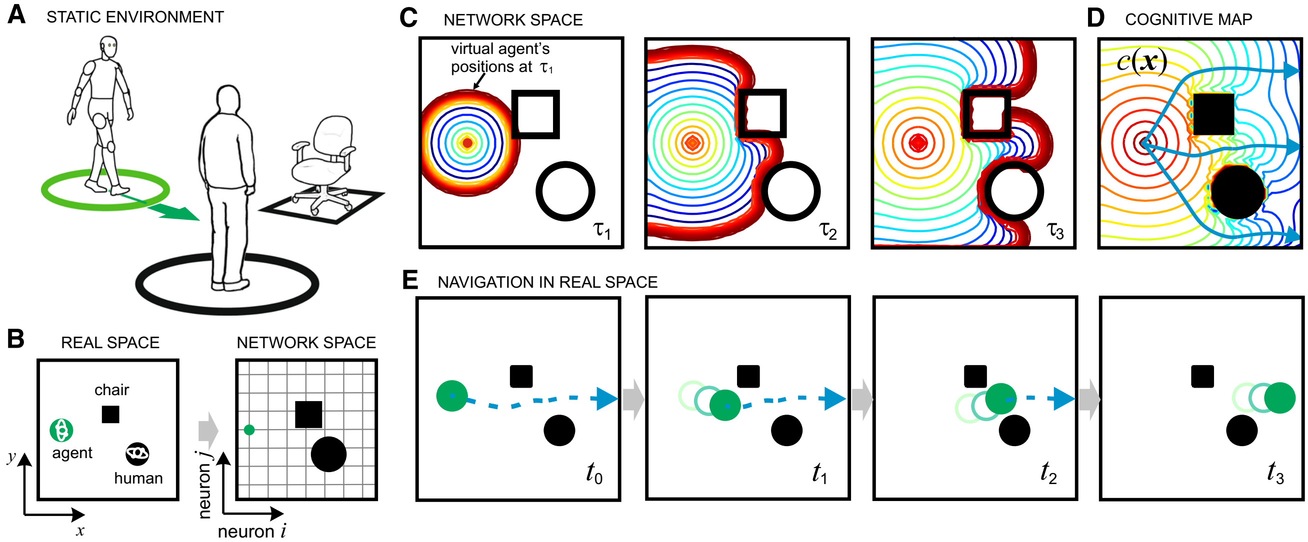

Fig. 1 Cognition through compact cognitive maps in a *static environment*. **a** A humanoid agent (*green circle*) walks avoiding collisions with a static human (*black circle*) and a chair (*black square*). **b** The situation is mapped from the real space (*left*) to the network space (*right*) described by a 2D neural lattice. **c** A wavefront propagating in the lattice simulates multiple agent's trajectories (three snapshots at mental time $\tau = \tau_{1,2,3}$). The front explores the environment and creates a gradient profile. **d** Final cognitive map with effective obstacles (in *black*). Going up the gradient, the agent can reach the target avoiding obstacles (*blue arrowed lines*). **e** Example of navigation. The agent follows one of the possible trajectories (superimposed frames with increasing *green* intensity correspond to progressively increasing time instants) (color figure online)

or a cognitive map (Fig. 1d, contour curves). Going up the gradient ∇c (transversally from red to blue curves), the agent can follow one of the virtual trajectories (Fig. 1d, blue arrowed curves). These trajectories ensure collision-free walking in the real space. Figure 1e illustrates one of the possible ways of navigation in this static environment.

2.2 Compact cognitive maps for dynamic situations

Figure 2a sketches a situation similar to that considered in Fig. 1a. However, now the human is going toward the chair, and therefore, the agent is in a *dynamic situation*, which challenges standard cognitive maps.

The core of the Prediction-for-CompAction paradigm relies on two basic elements: (i) prediction of the movements of objects and (ii) simulation of all possible agent's trajectories. A special neuron network matches these processes and generates a *compact* cognitive map. Functionally, this map is equivalent to a standard cognitive map, i.e., it is a static structure given by Eq. (2), which allows for tracing collision-free trajectories.

2.2.1 Prediction of object trajectory

To predict trajectories, we use a dynamic memory implemented in a so-called trajectory modeling neural network (TMNN, "Appendix A.1") (Villacorta-Atienza et al. 2010; Makarov and Villacorta-Atienza 2011).

A trajectory of a moving object (e.g., of the human in Fig. 2a) is a smooth function of time $s : \mathbb{R} \to \mathbb{R}^2$ that can be approximated by the Taylor expansion [similar to the spline method used by Kuderer et al. (2012)]:

$$s(t) = s(0) + s'(0)t + \frac{s''(0)t^2}{2} + O(t^3) \tag{3}$$

Such an assumption is valid, at least, for inanimate objects and, as we will see below, for humans under the AvUs paradigm. The TMNN predicts future object's locations within the model (3), $\tilde{\mathbf{s}}$, by iterating a linear map $(W^k \mathbf{s}_0)_{k \in \mathbb{N}}$, where W is a matrix describing couplings among neurons and $\mathbf{s}_0 = (s(0), s'(0), s''(0))^T$ is the vector of initial momenta of the object (i.e., position, velocity, and acceleration). Then,

$$\tilde{\mathbf{s}} = \tilde{\mathbf{s}}(\tau; \mathbf{s}_0) \tag{4}$$

is the trajectory in the network space D and mental time $\tau = kh$, where h is the time step and $k \in \mathbb{N}$.

For correct predictions, the TMNN must be trained, i.e., the connectivity matrix W must be properly tuned (Villacorta-Atienza et al. 2010). Once the learning is finished, the TMNN is ready to predict the movement of objects solely based on their positions acquired by the sensory system at the present ($t = 0$) and two time instants in the past ($t = -2h$ and $t = -h$). Such predictions are quite robust against sensory noise (Villacorta-Atienza and Makarov 2013).

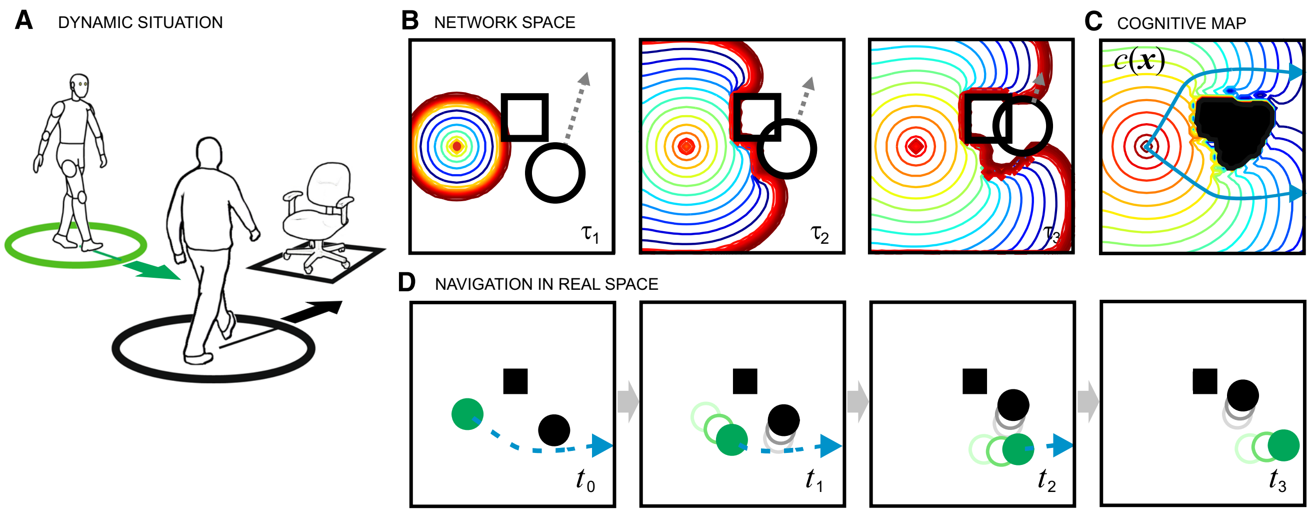

Fig. 2 Cognition through compact cognitive maps in a *dynamic situation*. **a** Same as in Fig. 1a, but now the human walks toward the chair. **b** Simulation of the agent's movements (wavefront) and matching them with obstacles' movements (human's trajectory, *dashed line*, is predicted by the TMNN). Collisions of the wavefront and virtual obstacles produce effective obstacles. **c** Compact cognitive map (the time dimension has been compacted) with static effective obstacle (*black*). Going up the gradient (*blue arrowed curves*) ensures collision-free walking. **d** Agent navigating in the real space (color figure online)

2.2.2 Simulation of agent's movements and matching them with predicted trajectories of objects

As in the static case, the CNN simulates all possible movements of the agent by a wavefront (Fig. 2b). However, since now the human moves in the environment, his trajectory, $\tilde{\mathbf{s}}(\tau)$, is predicted by the TMNN, and expected future positions are fed to the CNN (compare Figs. 1c vs. 2b). Collisions of the wavefront and virtual objects in the network space correspond to possible collisions of the agent with objects in the real space. In the CNN, these locations delimit effective obstacles (Fig. 2b; "Appendix A.3").

Once the network space has been explored, the dynamic situation is represented as a static map (Fig. 2c):

$$c = c(\mathbf{x}; \tilde{\mathbf{s}}), \quad \mathbf{x} \in D \tag{5}$$

The mobile (human) and immobile (chair) objects are replaced by the corresponding effective obstacles (joint black area). The gradient profile (contour curves from red to blue) contains a virtually infinite set of pathways that can be followed by the agent (Fig. 2c shows two representative examples):

$$\mathbf{d} = \mathbf{d}(\mathbf{x}; \nabla c) \tag{6}$$

Note that by simply avoiding static effective obstacles in $c(\mathbf{x})$, the agent avoids collisions with the human and the chair. Thus, the selected trajectory can be converted to motor actions, and the agent can navigate in the real space following the corresponding path, $d(t)$, (Fig. 2d).

Compact cognitive maps generalize the traditional concept of cognitive maps extending it to time-changing situations. They compress spatiotemporal information about where and what may happen into static structures. The decision-making scheme can be represented as a unidirectional chain:

$$s(t) \longrightarrow \tilde{\mathbf{s}}(\tau) \longrightarrow c(\mathbf{x}) \longrightarrow \mathbf{d}(\mathbf{x}) \longrightarrow d(t) \tag{7}$$

We note that at first glance the situation presented in Fig. 2a may be considered delusively simple. Some geometric-based methods could provide solutions to this path planing problem (Moussaid et al. 2011). However, in the presence of several moving objects, the problem becomes practically unsolvable for geometric algorithms or they would provide suboptimal solutions. Indeed, any trajectory deviation caused by avoidance of the first obstacle will induce changes in the configuration of possible collisions with the second obstacle, etc. Thus, the calculation diverges. Nevertheless, our neural network approach efficiently resolves navigation problems of practically arbitrary complexity.[1]

[1] Examples, simulations, and videos are available at http://www.mat.ucm.es/%7Evmakarov/research.php.

3 Cognitive navigation in social environments

The agent's behavior in an environment with other cognitive agents (humans, animals, and/or robots) will depend on whether or not it can elicit cooperation of other parties.

3.1 Machine avoids us (AvUs): noncooperative navigation

An AvUs agent expects no cooperation from humans and hence should move among people without disturbing them, "doing all the work" to navigate safely. Thus, an AvUs agent can consider humans as "inanimate" but moving objects, i.e., agent's decisions/actions, $d(t)$, produce *no feedback* to the human's trajectory, $s(t)$.

Figure 3a illustrates an AvUs agent approaching a human. The agent assumes noncooperative behavior of its human partner. Thus, the predicted human's movement depends uniquely on the initial conditions and will not be affected by the agent's decisions (given that the human will not suddenly change the trajectory). Thus, the basic Prediction-for-CompAction concept described in Sect. 2.2 [information flow (7)] is sufficient to resolve this situation.

Figure 3b shows the resulting compact cognitive map (the process of creation is analogous to that shown in Fig. 2b). The map contains a relatively big effective obstacle (black area) representing possible collisions with the human. Thus, the agent must steer around the effective obstacle to guarantee safety of the human and itself (Fig. 3b, blue arrowed curve).

3.2 Machine cooperates with us (CoUs): cooperative navigation

Let us now assume that humans are predisposed to cooperate with a properly looking and behaving humanoid agent. At the risk of a collision, both the human and the agent will make a step out from their straight trajectories thus helping each other (Fig. 4a).

The original Prediction-for-CompAction paradigm assumes the feedforward information flow (7). However, now the agent's decision (i.e., the chosen trajectory $d(t)$) will have a feedback to the movement of the human, $s = s(t; d(t))$, which closes the information loop through the environment, and the process becomes recursive:

$$\begin{tikzcd} s(t) \arrow[r] & \tilde{\mathbf{s}}(\tau) \arrow[r] & c(\mathbf{x}) \arrow[r] & \mathbf{d}(\mathbf{x}) \arrow[r] & d(t) \arrow[llll, bend left, "\text{environment}"] \end{tikzcd} \qquad (8)$$

This significantly challenges cognition. Indeed, the agent should simulate all possible decisions (i.e., it should create $c(\mathbf{x})$ and then $\mathbf{d}(\mathbf{x})$ on the basis of $\tilde{\mathbf{s}}(\tau)$). In turn, each decision, $d(t)$, will provoke a reaction of the human, thus changing $s(t)$ and hence the predicted trajectory $\tilde{\mathbf{s}}(\tau)$. Then, this new prediction should be taken into account to create new $c(\mathbf{x})$, etc.

To cope with recursive cognition, the agent must be able to model possible humans' actions using an internal loop:

$$\begin{tikzcd} s(t) \arrow[r] & \tilde{\mathbf{s}}(\tau) \arrow[r] & c(\mathbf{x}) \arrow[r] & \mathbf{d}(\mathbf{x}) \arrow[r] \arrow[ll, bend left, "\text{internal loop}"] & d(t) \arrow[llll, bend left, "\text{environment}"] \end{tikzcd} \qquad (9)$$

This internal loop should be flexible enough to account for the great variety of possible situations, according to the law of requisite variety by Ashby (1968).

3.2.1 Heuristic model of human perception-for-action

Earlier studies have shown that during walking humans perceive possible obstacles within certain visual angle and can estimate the time to collision (Schrater et al. 2000; Hopkins et al. 2004). If a CoUs agent crosses the reaction zone (Fig. 4b), then the human tends to cooperate. To describe phenomenologically the human's behavior, we adopt the following heuristics [similar to Guy et al. (2011)]:

– A human will cooperate in collision avoidance only if another agent crosses his reaction zone under the angle $\varphi_{\mathrm{crss}} \in (-5°, 5°)$ with the main visual axis (dashed line in Fig. 4b). Otherwise, no cooperation is expected, i.e., if $|\varphi_{\mathrm{crss}}| \geq 5°$, then the ,will be forced to behave like an AvUs one.
– Under cooperative behavior, to avoid collision humans change their velocity vector: $\mathbf{v}_{\mathrm{new}} = \mathbf{v}_{\mathrm{old}} + \mathbf{w}$, where $\mathbf{v}_{\mathrm{old}}$ is the initial velocity and $\mathbf{w}$ is the normal vector to $\mathbf{v}_{\mathrm{old}}$ ($\|\mathbf{w}\| = \frac{1}{2}\|\mathbf{v}\|, \mathbf{w}\cdot\mathbf{v} = 0$). The direction of $\mathbf{w}$ depends on the movement of the other agent.

We note that the second heuristic allows humans to keep the velocity in the target direction, which is useful, e.g., when moving in a group.

3.2.2 Compact cognitive maps under CoUs paradigm

In the network space, the agent's dimension is reduced to a single neuron (Fig. 4c, green point), while the lengths of the human's reaction, d, and personal, r, zones increase, accordingly. In agreement with the Prediction-for-CompAction

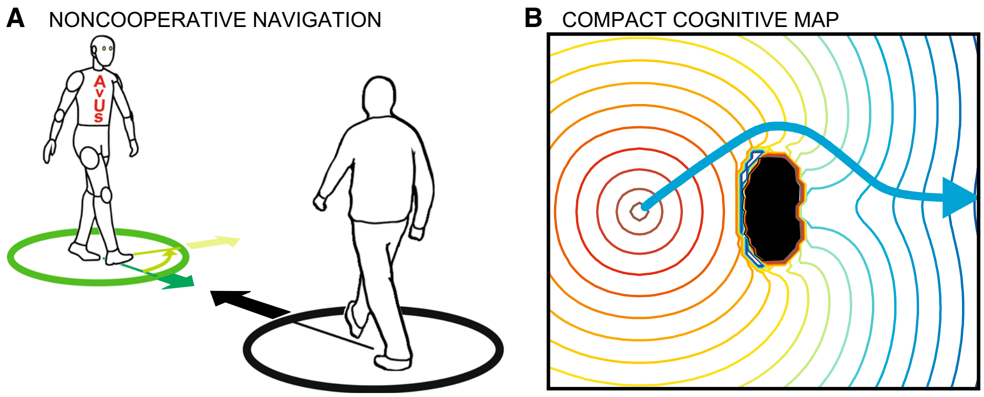

Fig. 3 Noncooperative navigation (AvUs paradigm). **a** An agent expects no cooperation from a human. At the risk of a collision, the agent steps away (*light green arrow*), while the human goes straightforward (*black arrow*). **b** Compact cognitive map. The effective obstacle (*black area*) forces the agent to steer its trajectory (*blue arrowed curve*) (color figure online)

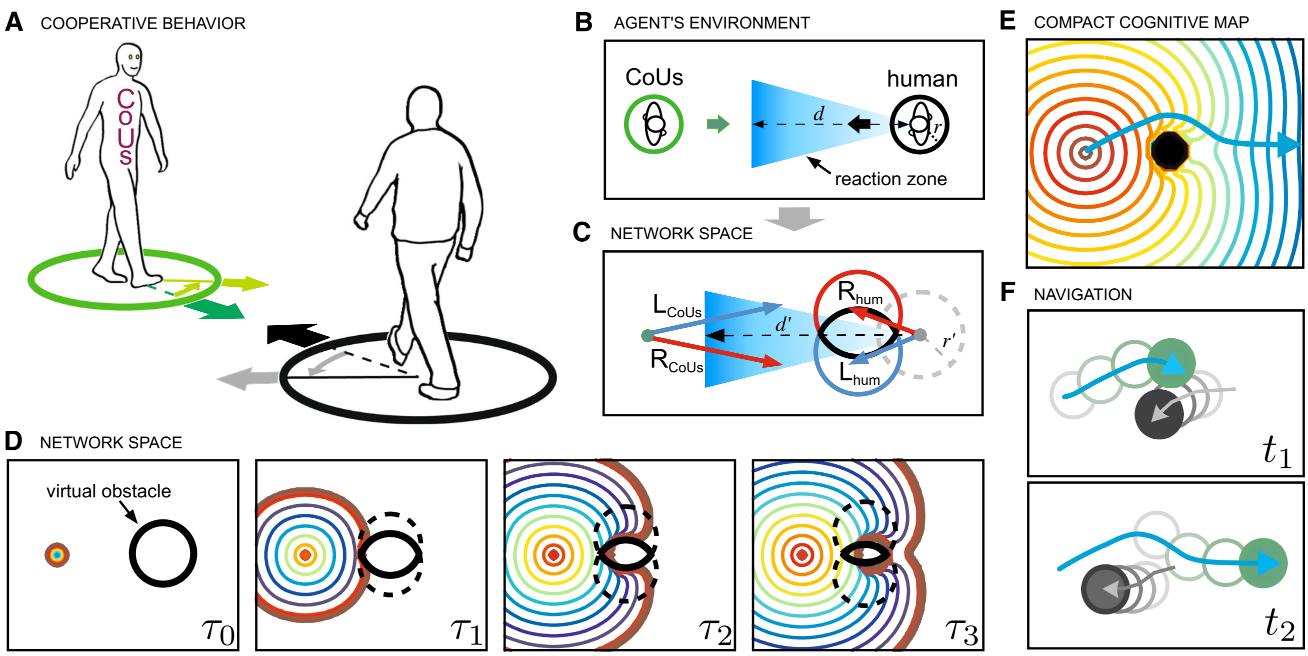

Fig. 4 Cooperative navigation (CoUs paradigm). **a** Same as Fig. 3a, but now the human cooperates in collision avoidance (*light gray arrow*). **b** Cooperation occurs only if the CoUs agent enters the human's reaction zone under a proper angle. **c** The agent can go either to the right, R_{CoUs}, or to the *left*, L_{CoUs}, expecting the human response R_{hum} and L_{hum}, respectively. The intersection of two personal zones forms a virtual obstacle to be avoided (area delimited by *black solid curve*). **d** The process of mental exploration of possible movements (note decreasing size of the virtual obstacle). **e** Cooperation reduces the effective obstacle in the compact cognitive map (compare to Fig. 3b) and enables more efficient navigation. **f** Example of navigation (superimposed frames with increasing color intensity correspond to progressively increasing time instants) (color figure online)

concept, a wavefront propagating in the network space simulates all possible agent's decisions. The wavefront will cross the human's reaction zone both from the right and from the left of the midline. This means that the agent can avoid the human either from the right or from the left (Fig. 4c, trajectories R_{CoUs} or L_{CoUs}, respectively). In the R_{CoUs} case, the human is expected to turn right, R_{hum}, whereas in the L_{CoUs} case, he will go to the left, L_{hum}. Since the human at the next time instant can occupy either of the two positions (Fig. 4c, red and blue circles), there appears uncertainty in the internal representation. The overlapping part of the corresponding personal zones (area delimited by black solid curve) creates a virtual obstacle that the agent must avoid. Note that the remaining part of each personal zone is avoided due to the human–agent cooperation.

Figure 4d illustrates the process of generation of a compact cognitive map. A wavefront propagates outwards the agent's initial position. At the beginning ($\tau = \tau_0$, no interaction with the reaction zone), the human is represented as a moving circular obstacle, according to his personal zone. When the wavefront reaches the reaction zone, the human starts cooperating. Thus, the circular obstacle is split into two circles separating along the time course. This way the agent simulates the human responses adjusting $\tilde{\mathbf{s}}(\tau)$ according to $\mathbf{d}(\tau)$ [internal loop in (9)]. Then, the virtual obstacle to be avoided is the intersection of these circles, which progressively diminishes (Fig. 4d, $\tau = \tau_{1,2,3}$).

Figure 4e shows the final compact cognitive map. It contains a single effective obstacle (black area), which is significantly smaller than that in the case of the AvUs agent (Figs. 4e vs. 3b). Thus, the CoUs agent takes advantage of the human cooperation and gets more room for walking (blue arrowed curve). Figure 4f shows an example of the CoUs navigation in the real space.

4 Navigation in crowd: performance gain due to cooperation

In order to study the performance gain and drawbacks provided by cooperation, we simulated navigation of AvUs and CoUs agents in different human environments. In each situation, we repeated internal simulations and decision making according to the two paradigms (Sect. 3). Then, we analyzed and compared the compact cognitive maps and agents' trajectories obtained from them.

4.1 Benefits of CoUs strategy: moving against cluttered pedestrian flow

Let us model a situation frequently observed in the real world: An agent goes along a corridor against a pedestrian flow coming out of a door (Fig. 5a, circles correspond to personal areas). We simulated the pedestrian flow as a cluttered bunch of humans going in the same direction. The pedestrians' velocities increase as they move away from the door. Such behavior is natural when people have room to move (Helbing and Molnar 1995). Besides, humans tend to optimize their trajectories and follow straight lines (Trautman and Krause 2010), unless they cooperate for avoiding collisions (Sect. 3.2.1).

Figure 5b shows the compact cognitive maps and paths to the door found by the AvUs and CoUs agents. The two paradigms differ significantly in the representation of the dynamic situation. In general, effective obstacles in the AvUs map are bigger than in the CoUs one (Fig. 5b, left vs. right). Few big effective obstacles in the CoUs map appear due to violation of the condition for cooperation (Sect. 3.2.1). In this case, the CoUs strategy is reduced to the AvUs one, and hence, we get similar effective obstacles in both maps. Thus,

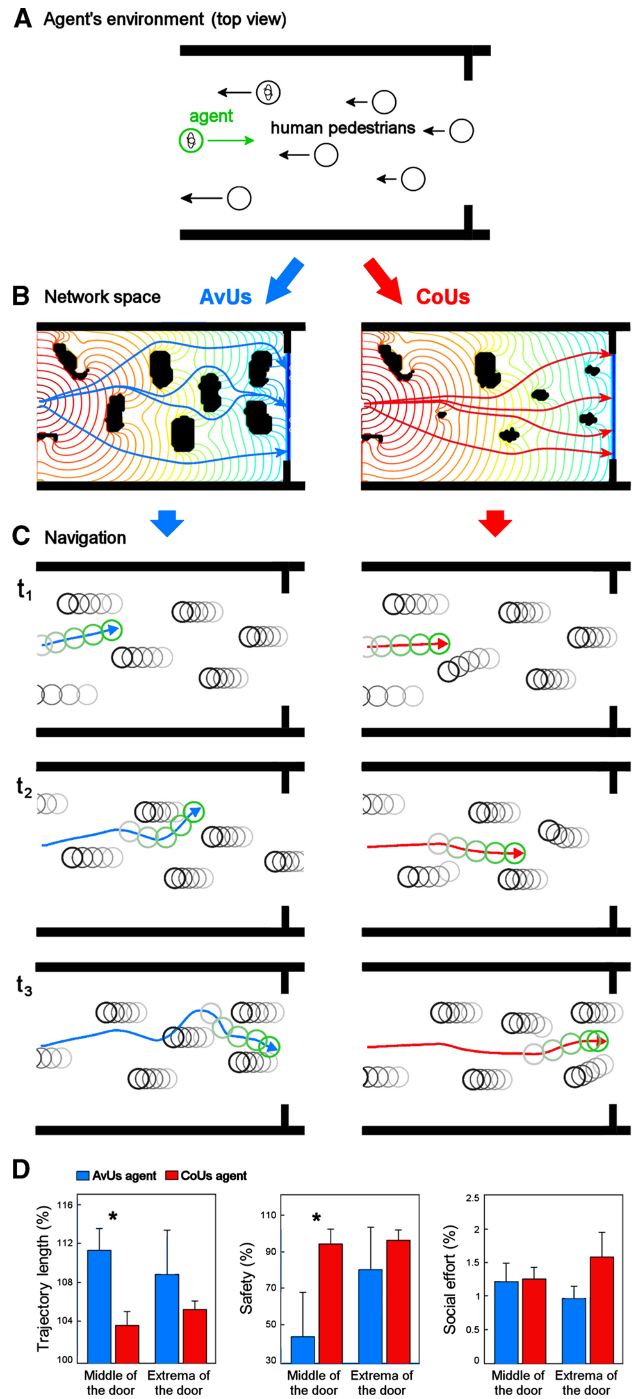

Fig. 5 Performance gain provided by cooperation in a cluttered crowd (CoUs vs. AvUs paradigm): **a** Initial situation. An agent (*green circle*) goes along a corridor to a door against pedestrian flow (*black circles*). *Arrows* indicate the pedestrians' velocities. **b** Compact cognitive maps created under noncooperative (*left*) and cooperative (*right*) behaviors. *Black areas* correspond to effective obstacles. *Arrowed curves* show possible paths to the door. **c** Examples of navigation of the AvUs (*left*) and CoUs (*right*) agents (superimposed frames with increasing color intensity correspond to progressively increasing time instants). **d** Measures of the navigation performance (mean and SD). *Stars mark* statistically significant difference, $p < 0.05$ (color figure online)

the CoUs agent gets more room for movement and, therefore, can plan more efficient trajectories to the door.

Figure 5c illustrates representative trajectories for both strategies. To avoid collisions, the AvUs agent steers abruptly on its path to the door (left subplots). In the same situation, the CoUs agent takes advantage of the human's cooperation. Its trajectory (right subplots) is significantly straighter. Note how pedestrians cooperate by deviating from straight lines and leaving more room to the CoUs agent.

To quantify the performance of the AvUs and CoUs strategies and to get deeper insight, we separated agent's trajectories into two subsets: trajectories reaching the middle part of the door (central door segment, $1/2$ width) and reaching the door's extrema (left and right door segments, $1/4$ width each). Then, we evaluated three performance measures (see "Appendix B").

Trajectory length Figure 5d (left) shows the mean normalized trajectory lengths for the AvUs and CoUs agents. As expected, the CoUs agent in general follows shorter paths to the door relatively to the AvUs. Moreover, the difference is statistically significant if the agents target the door center [$p = 0.04$; here and below, we used t test (Winter 2013)], i.e., if they try to take the shortest way to the door, which favors cooperation and provides higher benefit. If the agents decide to go around the crowd to either of the door extrema, the difference between the AvUs and CoUs strategies decreases ($p = 0.254$). It occurs due to low chances of cooperation with pedestrians on such a route.

Trajectory safety In many real situations, the choice of a particular trajectory does not solely rely on its length, but also includes safety as an important factor. We defined the navigation safety as the fraction of the trajectory passing sufficiently far away from the effective obstacles, i.e., keeping the distance longer than some critical value. Figure 5d (middle) shows the trajectory safety for the AvUs and CoUs agents. Surprisingly, cooperation provides better navigation safety, especially for trajectories going to the door center ($p = 0.027$). We note that this is not due to the trajectory straightness (length) but due to the reduced size of effective obstacles (Fig. 5b).

Mean social effort Until now, we considered the performance gain of the CoUs strategy from the agent's viewpoint. However, cooperation also implies elongation of trajectories of other pedestrians. Thus, as expected, the CoUs strategy provides benefits to the agent at the cost of inconvenience for other pedestrians. Such a situation may be socially unstable. Thus, we measured the impact of cooperation on the "society" as the mean trajectory elongation averaged over all pedestrians (including the agent). In the case of targeting the door center, the CoUs strategy produces no additional load

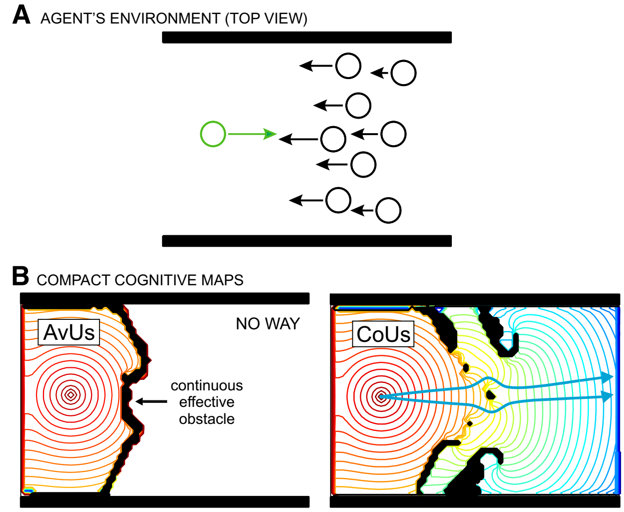

Fig. 6 Cooperation can help solve complex navigation problems. **a** Initial situation. An agent (*green circle*) goes against a compact group of people. *Arrows* indicate pedestrians' velocities. **b** Compact cognitive maps created by the AvUs (*left*) and CoUs (*right*) agents. In the noncooperative AvUs case, an effective obstacle closes the corridor impeding navigation. In the cooperative CoUs case *blue arrowed curves* indicate two possible pathways (color figure online)

to the social effort (Fig. 5d, right, $p = 0.9$). However, the social load increases (not significantly, $p = 0.19$) for trajectories going to the door extrema. Thus, in the case of targeting the door center (natural for human's behavior), cooperation provides benefits in terms of the trajectory length (time and energy consumption) and safety with no additional load to the society effort.

4.2 CoUs strategy can be necessary for navigation in dense crowd

Let us now show that besides being convenient cooperation can be necessary for successful navigation. Figure 6a illustrates a situation with an agent going against a compact group of pedestrians occupying the entire width of the corridor. The group leaves no room to an AvUs agent. Indeed, the compact cognitive map provides no solution (Fig. 6b, left), since the wavefront (simulating the possible agent's movements) cannot "leak" through pedestrians. This happens since under mental simulation the agent's personal area is added to the areas occupied by other pedestrians and holes among them disappear. The CoUs paradigm elicits cooperation. Humans move away and leave room to the agent, enough to steer among them (Fig. 6b, right). Thus, cooperation may be not only beneficial but also necessary to reach the goal.

4.3 Abuse of cooperation may penalize CoUs strategy

Figure 7a shows a situation with a narrow door, which forces pedestrians line up in a chain. Then, the AvUs agent must go around the crowd since effective obstacles completely occupy the middle part of the cognitive map (Fig. 7b, left). In the same situation, the CoUs agent can go either around or through the crowd. Cooperation leads to significantly smaller effective obstacles leaving holes among them (Fig. 7b, right). Figure 7c illustrates representative examples of trajectories performed by both agents. The CoUs agent "pushes" pedestrians from its way and reaches the door following practically a straight line.

Figure 7d provides statistical properties of the AvUs and CoUs strategies. Since the door is narrow in this case, we made no distinction between the center and extrema. The mean trajectory length is lower under cooperation, as expected (Fig. 7d, left). However, this decrease is not significant ($p = 0.19$). The navigation safety decreases under cooperation, but again not significantly (Fig. 7d, middle, $p = 0.16$). This occurs because trajectories can go through the pedestrian chain and hence pass nearby humans, which increases chances of collision. Noteworthy, the cooperative strategy produces a statistically significant load to the social effort (Fig. 7d, right, $p = 0.02$), which is not acceptable for the robot behavior.

Thus, the cooperative CoUs strategy can lose against the noncooperative AvUs navigation. This paradoxical situation may occur in some particular spatial arrangements of the crowd. Indeed, the chain arrangement (Fig. 7a) forces many pedestrians to stand away and the CoUs agent "squeezes" through the crowd. Humans usually do not follow such sociopathic behavior, since it is socially not acceptable and increases the risk of collisions with many pedestrians. Therefore, abuse of cooperation by the CoUs agent may be inconvenient.

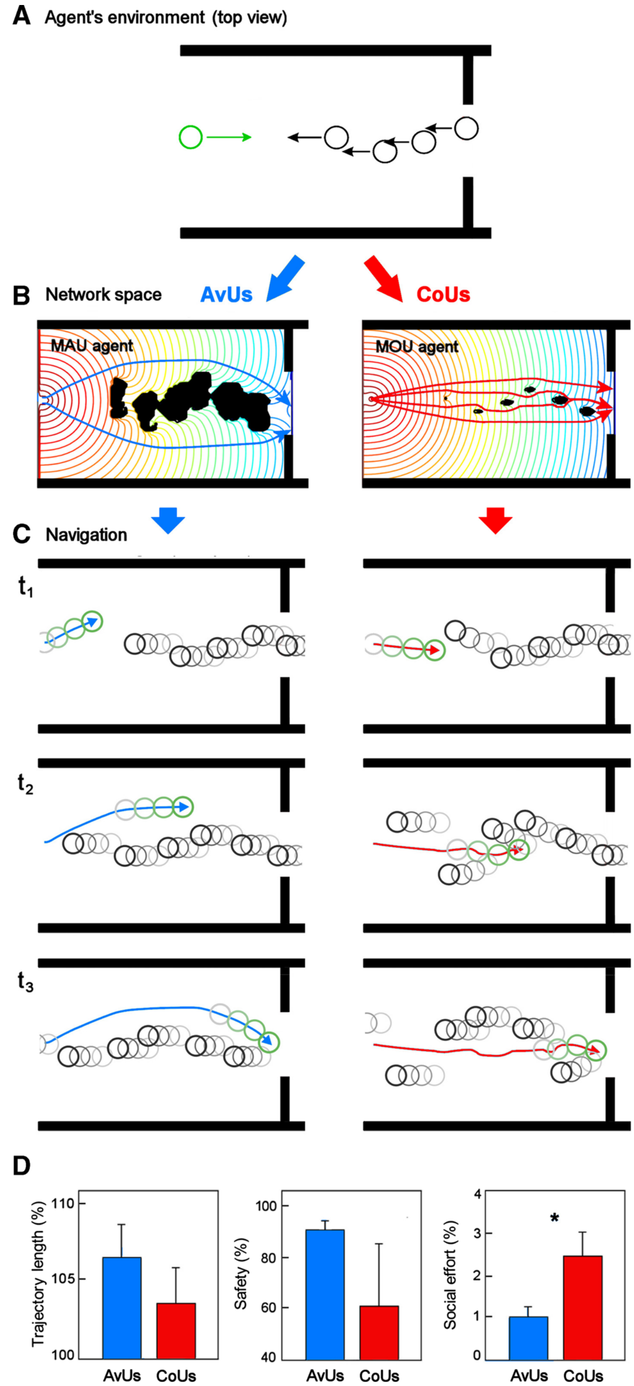

Fig. 7 Cooperative CoUs strategy can lose against non-cooperative AvUs navigation. **a** Initial situation. An agent (*green circle*) goes along a corridor to a narrow door against a lineup pedestrian flow (*black circles*). *Arrows* indicate the pedestrians' velocities. **b** Compact cognitive maps created by the AvUs (*left*) and CoUs (*right*) agents. *Black areas* correspond to effective obstacles. *Arrowed curves* show possible paths to the door. **c** Examples of navigation of the AvUs (*left*) and CoUs (*right*) agents (superimposed frames with increasing color intensity correspond to progressively increasing time instants). **d** Measures of the navigation performance (mean and SD, *star marks* statistically significant difference) (color figure online)

5 Discussion

Artificial cognition largely deals with the comprehension of relationships among elements in the environment. Nowadays the theoretical concept of cognitive maps, as a mean for understanding static situations, received strong experimental support (Schmidt and Redish 2013). However, in dynamic situations, spatial relationships among objects evolve in time. Therefore, the corresponding cognitive map should also change in time, which contradicts the very concept of a map.

In this work, first we revisited the question of how the theory of compact internal representation (Villacorta-Atienza et al. 2010) can generalize the concept of cognitive maps upon dynamic situations. The used neural architecture consists of two coupled neural networks. A recurrent neural network predicts positions of the obstacles for $t > 0$. These data are mapped into the other 2D neuronal lattice that simulates what will happen if the agent will take this or that trajectory. A wavefront propagating over the lattice collides with obstacles provided by the first network and forms a static potential field surrounding "islands" of effective obstacles (Fig. 2). Thus, a dynamic situation can be "mentally" represented as a static structure similar to a classical map. We called this process Prediction-for-CompAction (PfCA). The obtained compact cognitive map serves as a dynamic GPS that enables navigation avoiding collisions both with static and moving obstacles. In this work, we revised the lattice dynamics [earlier proposed by (Villacorta-Atienza et al. 2010)] in such a way that the new neural network implementation makes no *a priori* distinction between obstacles and targets. Thus, new compact cognitive maps are more universal, since the same object in a map can be assigned as a target or as an obstacle and the agent can plan a chasing or escaping actions.

The human behavior differs significantly from the behavior of inanimate but moving objects. Therefore, cognition of situations involving humans brings another dimension of complexity. In this case, the agent's actions depend on cognitive decisions made by humans and vice versa. Thus, the cognitive process becomes recursive (Dennett 1987). In this article, within the PfCA paradigm, we addressed the first level of recursion. The agent assumes that its human partner under the risk of a collision will deliberately change his trajectory. Thus, the original concept of the trajectory prediction, based solely on initial conditions (Villacorta-Atienza et al. 2010), should be corrected for humans. Then, we equipped the agent with an internal heuristic model of the human behavior, similar to that proposed by Guy et al. (2011). The model assumes that a human changes the velocity vector once the agent enters his reaction zone ($\mathbf{v}_{\text{new}} = \mathbf{v}_{\text{old}} + \mathbf{w}$). This prediction is introduced into the 2D neuronal lattice and, as in the inanimate case, a wavefront explores all possible agent's actions and generates a compact cognitive map describing recursive cognition (Fig. 4). Thus, the novel PfCA theory allows for coexistence of cognitive agents and cooperation emerges as a product of the neural network dynamics.

In order to investigate the plausibility and performance of the proposed theory, we simulated navigation of an agent in different social environments. Two behavioral paradigms have been tested: "machine avoids us" (AvUs) and "machine cooperates with us" (CoUs). For navigation under either of the paradigms, the agent builds a compact cognitive map. The map structure (position and size of effective obstacles) depends on the assumed human behavior, either cooperative or noncooperative. We have shown that the CoUs strategy sizably reduces effective obstacles comparing to the AvUs map (Fig. 5). Therefore, in the same situation, a CoUs agent gets more room for movement than an AvUs one. Thus, in many realistic situations, including navigation in cluttered pedestrian flows, a CoUs agent can choose significantly shorter paths to the target (and hence spend less time and energy). Moreover, the navigation safety (i.e., chances to avoid accidental collisions) increases under cooperation.

Cooperative navigation usually forces pedestrians turn aside from their straight courses. This inevitably elongates trajectories of all partners. The acceptable degree of cooperation in human society and the ensuing performance drop depend on many factors (e.g., cultural, emotional, or environmental). Then, the CoUs strategy will be socially acceptable if its cost to human pedestrians will be reasonable (similar to human cooperation). Thus, we measured the mean impact of the CoUs strategy on the society, i.e., the mean elongation of the trajectories averaged over all pedestrians including the agent. We have shown that in unstructured, low-dense pedestrian flows (such as in Fig. 5), the CoUs agent produces no additional load to the society effort. Thus, a PfCA equipped artificial agent can behave as "one of us".

Another advantage of the CoUs strategy appears in extremely dense environments. It may happen that a group of people leaves no room for passing through (Fig. 6). Then, the AvUs strategy provides no way to the target, whereas the cooperative behavior of people helps a CoUs agent find a path through the bunch of people. It is worth noting that the PfCA allows detecting situations with no solution, as, e.g., in Fig. 6 under AvUs. Thus, we can estimate chances of success without taking unnecessary risks of pursuing unreachable targets. This is a key attribute of the global decision making, distinguishing evolved animals from simple living beings (Villacorta-Atienza and Makarov 2013).

Though cooperation is usually beneficial, humans do not always exploit it. We provided an example of such a situation: an agent going against a dense, chain-like pedestrian flow coming from a narrow door (Fig. 7). We have shown that in this situation, the CoUs strategy loses against the noncooperative AvUs navigation. This paradoxical result occurs due to the spatial arrangement of the crowd, which constrains many pedestrians to give pass to the CoUs agent squeezing its way through the crowd. Humans usually avoid such sociopathic decision, since it highly increases the risk of collisions with many pedestrians and is socially reproachable. Therefore, in certain situations, cooperation may be inconvenient. Then, prior to taking any action, an agent equipped with compact cognitive maps could "mentally" evaluate the advantages and risks of the CoUs and AvUs behaviors and adopt the correct decision.

In conclusion, the proposed neural network architecture provides cognitive skills necessary for versatile and efficient navigation in social environments. The introduced AvUs and CoUs navigation strategies are complementary, and their use depends on the context. An advanced artificial agent should test both of them and select the appropriate for each situation. Moreover, the PfCA theory enables high-level cognitive abilities like learning and memory, which is a sensible challenge for any artificial system (Villacorta-Atienza and Makarov

2013). Then, successful experiences can be learnt and transformed into efficient automatic-like behaviors. The proposed approach has been tested focusing on the agent's internal simulation and decision making. Its capacity in dealing with uncertainty and deviation of humans from the assumed cooperative or noncooperative behavior is an open question left for further studies.

Appendix A: Neural network implementing compact cognitive maps

The main principles and details of the compact internal representation (CIR) have been discussed elsewhere (Villacorta-Atienza et al. 2010; Villacorta-Atienza and Makarov 2013). Briefly, the CIR is generated by a causal neural network (CNN) that receives as an input locations of all objects in the arena predicted by the trajectory modeling neural network (TMNN). The joint network dynamics forms effective static elements (e.g., effective obstacles) and a potential field c in the network space D, which constitute a compact cognitive map. Then, the map can be used to trace trajectories to a target.

A.1 Trajectory modeling

The TMNN implements a dynamic memory by means of a recurrent neural network (Makarov et al. 2008). It models the object trajectory $s(t)$ by a quadratic polynomial [see Eq. (3)]. In 2D space, we have two components $s(t) = (x(t), y(t))$, each of which is modeled by one TMNN. Each TMNN consists of three recurrently coupled neurons with external input $\xi(k) \in \mathbb{R}^3$ and output $\eta(k+1) \in \mathbb{R}^3$, where $k = 0, 1, 2, \ldots$ is the discrete *mental* time (Villacorta-Atienza et al. 2010). The TMNN dynamics is given by

$$\eta(k+1) = \begin{cases} \xi(k), & \text{if } |\xi(k)| > \delta \\ W\eta(k), & \text{otherwise} \end{cases} \tag{10}$$

where $W \in \mathcal{M}_{3\times 3}(\mathbb{R})$ is the coupling matrix and δ is the tolerance constant ($\delta = 10^{-6}$).

The TMNN operates in two phases: learning and prediction. Under learning, the TMNN receives at the input an object trajectory $\xi(k) = (x(k), v(k), a(k))^T$, where $x(k)$, $v(k)$, and $a(k)$ are the position, velocity, and acceleration of the object, respectively. Then, the interneuronal couplings are updated according to:

$$\begin{aligned} W(k+1) = W(k)(I - \epsilon\xi(k-1)\xi^T(k-1)) \\ + \epsilon\xi(k)\xi^T(k-1) \end{aligned} \tag{11}$$

where $\epsilon > 0$ is the learning rate. The learning process (11) converges, given that ϵ is small enough [the critical value ϵ^* is given by Makarov et al. (2008)].

Once the learning is deemed finished, the TMNN can predict trajectories. The object initial moments $\xi(0) = (x(0), v(0), a(0))$ are sent to the TMNN at $k = 0$ and then $\xi(k) = 0$ for $k > 0$. On the output, we get the predicted trajectory: $\eta(k) = W^k\xi(0)$. The TMNN performance has been tested experimentally using as moving objects mobile robots (Villacorta-Atienza and Makarov 2013).

A.2 Causal neural network

The CNN is a 2D lattice of FitzHugh–Nagumo neurons (80×80 cells in numerical experiments). The lattice dynamics is given by:

$$\begin{aligned} \dot{r}_{ij} &= q_{ij}\left(f(r_{ij}) - z_{ij} + d\Delta r_{ij}\right), \quad (i, j) \in D \\ \dot{z}_{ij} &= \varepsilon(r_{ij} - 7z_{ij} - 2) \end{aligned} \tag{12}$$

where r_{ij} and z_{ij} are the membrane potential and recovering variable of the (i, j)th neuron, respectively. Dots represent derivatives with respect to the mental time $\tau = hk$, Δ is the discrete Laplacian, and $f(r)$ is a cubic like nonlinear function. The system (12) is considered with Neumann boundary conditions. In numerical experiments, we used $d = 0.2$, $\varepsilon = 0.04$, and $f(r) = (-r^3 + 4r^2 - 2r - 2)/7$. The function $q_{ij}(\tau)$ describes effective objects and will be discussed in Sect. A.3.

At the beginning, all cells are set at rest ($r_{ij}(0) = z_{ij}(0) = 0$) except one. The neuron (i_a, j_a) corresponding to the agent's location has no dynamics $q_{i_a j_a} = 0$ and hence $r_{i_a j_a}(\tau > 0) = r_{i_a j_a}(0) = 5$ (we remind that in the network space D, the agent is reduced to a single cell).

A.3 Compact cognitive map and effective objects

The TMNN predicts movements of the obstacles and targets in the environment, while the CNN matches this information with the process of simulation of agent's movements.

A wavefront propagating from the agent position is generated in the CNN (see Fig. 2b). It switches cells to upstate. The time $\tau = c$ when the cell (i, j) crosses a threshold $(r_{ij}(\tau = c) = r_{\text{th}})$ is stored. Thus, behind the wavefront, we obtained a potential field $\{c_{ij}\}$ [see also Eq. (2)].

Let $\mathcal{B}(k)$ be a set of cells $\{(i, j)\} \in D$ occupied by obstacles and targets at the mental time k. Then, we define the following iterative process:

$$\Omega(k) = \Omega(k-1) \cup \delta\Omega(k), \quad k = 1, 2, \ldots; \quad \Omega(0) = \emptyset \tag{13}$$

where

$$\delta\Omega(k) = \{(i, j) \in D : \quad r_{ij}(kh) \in [1, 2], \quad (i, j) \in \mathcal{B}(k)\}$$

The set $\Omega(k)$ describes effective objects (obstacles and targets) in the network space D. It is dynamically created as the wavefront explores D. The set grows (i.e., $\delta\Omega(k) \neq \emptyset$) if the wavefront touches an object at $\tau = kh$. Then, we define the function $q(\tau)$ used in Eq. (12) as:

$$q_{ij}(\tau) = \begin{cases} 0, & \text{if } (i, j) \in \Omega(k) \\ 1, & \text{otherwise} \end{cases}$$

The cells in $\Omega(k)$ will exhibit no dynamics, i.e., the effective objects are *static* and the wavefront slips around them (Fig. 2b, panels $\tau_{2,3}$).

Once the exploration of D has been finished, the created CIR of the dynamic situation represents a compact cognitive map (Fig. 2c). It contains spatial relationships (a potential field c) structured by static effective objects. These effective objects contain critical information about possible collisions of the agent and obstacles (to be avoided) or targets (to be pursued).

A.4 Trajectory tracing

To obtain a trajectory, we use the gradient descent method. Since compact cognitive map does not distinguish between obstacles and targets, we should designate one (or several) of the effective objects in the map to be a target. Then, we start from some point at the effective target and go down the gradient $\gamma_{k+1} = \gamma_k - \nabla c$. The obtained trajectory ends at the agent's location (the deepest part of the potential).

We note that by construction the potential $c(i, j)$ has no local minima, and hence, a solution always exists (Fig. 2c, arrowed curves).

Appendix B: Measures of navigation performance

B.1 Trajectory length

Let us assume that an agent should move from a starting point $p_\mathrm{A} = (x_\mathrm{A}, y_\mathrm{A})$ to a target point $p_\mathrm{T} = (x_\mathrm{T}, y_\mathrm{T})$. On its route, the agent follows a trajectory given by vertices $\{p_i\}_{i=0}^{N}$, such that $p_0 = p_\mathrm{A}$ and $\|p_N - p_\mathrm{T}\| < \epsilon$ (ϵ is the navigation tolerance and $\|\cdot\|$ is the Euclidean norm). Then, we quantify the trajectory length relative to the shortest straight path to the target:

$$L = \frac{1}{\|p_\mathrm{A} - p_\mathrm{T}\|} \sum_{i=1}^{N} \|p_i - p_{i-1}\| \tag{14}$$

The closer the normalized length L to 1, the shorter the trajectory and the higher the navigation performance (i.e., faster arrival and smaller energy consumption).

B.2 Trajectory safety

Let d_crt be a critical distance between the agent and obstacles at which the navigation is considered safe. Then, we define the measure of safety of a trajectory Γ as the ratio:

$$S = 1 - \frac{\mathrm{card}(\delta\Gamma)}{\mathrm{card}(\Gamma)} \tag{15}$$

where

$$\delta\Gamma = \{p = (x, y) \in \Gamma : \quad D(p, \Omega) < d_\mathrm{crt}\}$$

where Ω is the final set of effective obstacles, defined by (13), and

$$D(p, \Omega) = \inf_{\tilde{p} \in \Omega} \|p - \tilde{p}\|$$

B.3 Mean social effort

We define the social effort as the average elongation of the normalized trajectories of all pedestrians, including the agent:

$$E = \frac{1}{M + 1} \sum_{i=0}^{M} (L_i - 1) \tag{16}$$

where M is the number of pedestrians and L_i is the trajectory length (14) of the ith pedestrian ($i = 0$ corresponds to the agent). The higher the value of E, the greater the effort of the society members to navigate to their goals.

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