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Synchronization of Heteroclinic Circuits Through Learning in Chains of Neural Motifs ⋆

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Abstract: The synchronization of oscillatory activity in networks of neural networks is usually implemented through coupling the state variables describing neuronal dynamics. In this study we discuss another but complementary mechanism based on a learning process with memory. A driver network motif, acting as a teacher, exhibits winner-less competition (WLC) dynamics, while a driven motif, a learner, tunes its internal couplings according to the oscillations observed in the teacher. We show that under appropriate training the learner motif can dynamically copy the coupling pattern of the teacher and thus synchronize oscillations with the teacher. Then, we demonstrate that the replication of the WLC dynamics occurs for intermediate memory lengths only. In a unidirectional chain of N motifs coupled through teacher-learner paradigm the time interval required for pattern replication grows linearly with the chain size, hence the learning process does not blow up and at the end we observe phase synchronized oscillations along the chain. We also show that in a learning chain closed into a ring the network motifs come to a consensus, i.e. to a state with the same connectivity pattern corresponding to the mean initial pattern averaged over all network motifs.

Keywords: Learning, Synchronization, Network motifs, Nonlinear dynamics.

1. INTRODUCTION

Complex large-size biological, ecological, and engineering networks can be frequently decomposed into relatively small network motifs, i.e. network patterns that occur significantly more frequently than in a random graph. Then, the study of the network structural properties can be addressed through investigation of universal classes or building blocks of recurrent network motifs (Milo et al., 2002). How one network motif can dynamically replicate the internal structure and the behavior of another one is an open problem.

Traditionally, synchronization of oscillations in network systems involves transmission of signals (energy) from one network element to another. For example, in neural networks synaptic couplings may convey electrical or chemical signals from one motif to another, which frequently promotes synchronization (Abarbanel et al., 1996). However, synchronization can also be achieved through a learning process. In this case there is no direct link between two networks. Instead, the information transfer is attained through observation of the teacher dynamics and by consecutive tuning of the connectivity pattern in the learner. Although such kind of synchronization is abundant in real world (e.g. children can learn movements shown by a teacher), its study from a dynamical systems point of view has attracted relatively little attention.

Earlier it has been shown that oscillations in network systems can emerge from a stable heteroclinic channel (Ashwin and Chossat, 1998; Ashwin and Field, 1999). In a neural network consisting of more than two competing neurons with unbalanced inhibitory connections, one may observe a situation when each neuron sequentially becomes a winner (i.e. strongly activated) for a limited time interval and then another neuron takes over the leadership. Dynamically such an operating mode, called winner-less competition (WLC), occurs in a vicinity of heteroclinic trajectories connecting saddle equilibria in a loop. Under certain conditions, the heteroclinic loop can be stable and then in the presence of a weak noise the trajectory will wander from one saddle to another (Cohen and Grossberg, 1983; Rabinovich et al., 2001; Varona et al., 2002).

In this work we propose a learning rule which allows one neural network, acting as a teacher, to impose the same heteroclinic circuit in another "learner" network. As a result, in the learner there appear WLC oscillations synchronized in phase with the oscillations of the teacher. We study how the information on the connectivity structure is replicated in a chain of network motifs. The proposed learning rule includes memory effects, i.e. the learner integrates over time the incoming information. We

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then provide conditions necessary for replication of the connectivity patterns in a learning chain of network motifs and also describe a "consensus" behavior on a ring.

2. MODEL DYNAMICS: SYNCHRONIZATION BY LEARNING

Figure 1A shows the architecture of a network motif composed of three recurrently coupled neurons. For the sake of simplicity we assume that the coupling strengths β are hard coded (i.e. fixed), while $\alpha = \{\alpha_k\}_{k=1}^3$ can be changed. Further on we will consider a unidirectional chain of such motifs (Fig. 1A). At the beginning the couplings α are arbitrary distributed among network motifs, and hence the motifs exhibit different dynamics. The purpose of learning is to replicate the coupling pattern α from the teacher and synchronize oscillations along the chain.

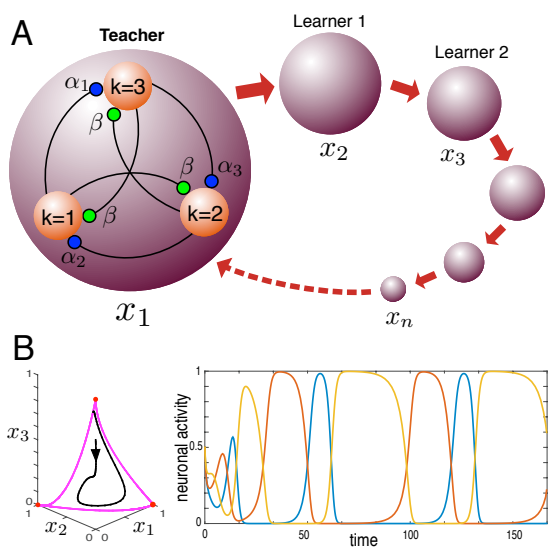

Fig. 1. The model. A) Network motifs consist of three recurrently coupled neurons each. Motifs are linked in a unidirectional learning chain. No direct coupling among the state variables exists. Instead, learner n adjusts its connectivity pattern to that of motif $n-1$ and thus synchronizes oscillations. If the last motif is linked to the first one we get a ring chain without leader. B) Winner-less dynamics in the phase space of a single motif (left) and time evolution of the neuronal activity (right). Blue, red, and yellow curves correspond to neurons 1, 2, and 3, respectively [$\alpha = (0.1, 0.6, 0.8)$].

2.1 Heteroclinic circuit: Winner-less dynamics

The network of motifs is organized by the teacher-learner principle. The governing equation of the teacher is given by the generalized Lotka-Volterra system

$$\dot{x} = x \odot (1 - \rho x) + \eta(t) \tag{1}$$

where $x(t) \in \mathbb{R}_+^3$ describes the activation state of three neurons at time t (Fig. 1B); $\odot$ stands for the Hadamard product; $\eta(t) \in \mathbb{R}^3$ is a Gaussian uncorrelated white noise with the mean 2e-5 and the standard deviation 1.5e-3; and the matrix $\rho \in \mathcal{M}_{3\times 3}(\mathbb{R}_+)$ accounts for local couplings among the neurons:

$$\rho = \begin{pmatrix} 1 & \alpha_2 & \beta \\ \beta & 1 & \alpha_3 \\ \alpha_1 & \beta & 1 \end{pmatrix}$$

Given that the following conditions are satisfied

$$\alpha_k < 1 < \beta, \quad \prod_{k=1}^{3}(1-\alpha_k) < (\beta - 1)^3, \tag{2}$$

it has been shown (Afraimovich et al., 2004) that in the system (1) there exists a globally stable heteroclinic circuit (Fig. 1B, left). Further on we will assume that $\beta > 2$ ($\beta = 2.8$ in numerical simulations). Then, condition (2) will be satisfied for any $\alpha_k < 1$. We will use double notation: index k will be used for all intra-motif variables, whereas index n will denote the motif number in the learning chain.

2.2 Synchronization by learning

Let us now consider a learning chain of network motifs (Fig. 1A). Since the learning is unidirectional, we can consider a pair teacher-learner, i.e. motifs n and $n-1$ will be referred to as a learner and a teacher, respectively.

Under phase synchronization by learning we understand the situation when independently on the teacher coupling pattern α_{n-1} and the initial conditions $[x_{n-1}(0), x_n(0), \alpha_n(0)]$ after some transient the following inequality is satisfied

$$|\phi_{n-1}(t) - \phi_n(t)| < M \tag{3}$$

where ϕ_{n-1} and ϕ_n are the oscillatory phases in the teacher and in the learner, respectively, and M is a constant.

Without loss of generality, we can assume that each teacher motif has a fixed coupling structure. At the beginning, the connectivity pattern in the learner $\alpha_n(0)$ is taken arbitrary from uniform distribution over $(0,1)^3$. Then the purpose of learning is to "copy" the coupling structure and consequently to synchronize oscillations in the learner with the teacher.

Since the teacher network cannot change the learner state $x_n(t)$ directly, but through the coupling strengths $\alpha_n(t)$ only, during the learning we expect:

$$\lim_{t\to\infty} \|\langle \alpha_n \rangle_T(t) - \alpha_{n-1}\|_2 = 0 \tag{4}$$

where

$$\langle u \rangle_T(t) = \frac{1}{T}\int_{t-T}^{t} u(s)\,\mathrm{d}s$$

denotes the time averaging operator over period T. Then, fulfillment of (4) ensures (3). In numerical simulations the learning will be deemed finished if the norm in (4) falls below a tolerance value $0 < \delta \ll 1$ for some t^*.

2.3 Learning rule

We will employ a Hebb-like rule for learning. First, we introduce a functional:

$$g(u(t)) = u(t) \odot \frac{1}{\tau}\int_{t-\tau}^{t} u(s)\,\mathrm{d}s \tag{5}$$

where $\tau \geq 0$ is a constant describing the memory length. The function $g(x_{n-1})$ represents the cumulative activity

of the neurons in the teacher network. Then, we can introduce the following learning rule:

$$\dot{\alpha}_n = \epsilon[g(x_{n-1}) - g(x_n)] \tag{6}$$

where $\epsilon > 0$ is the learning rate and the term in brackets is the error function described as teacher forcing based on the classical delta rule (Makarov et al., 2008). Note that in general the learning error $E = g(x_{n-1}) - g(x_n)$ can be an oscillatory function of time, even at $t \to \infty$. Therefore, we will say that the learning given by equation (6) is successful if $\lim_{t\to\infty}\langle E\rangle_T(t) = 0$

3. EFFECT OF MEMORY ON SYNCHRONIZATION

The memory time constant τ plays a significant role. Earlier we have proven that the learning process (6) converges to a globally stable one-parametric manifold for $\tau = 0$ (Calvo et al., 2016). To illustrate such behavior, we simulated the learning process in a chain of two network motifs: a teacher and a learner.

Figure 2A shows five examples of trajectories converging to a stable manifold in the phase space of the learner couplings ($\tau = 0$). Thus, the learning leads to a mismatch in the connectivity structure, which in turn causes synchronization failure.

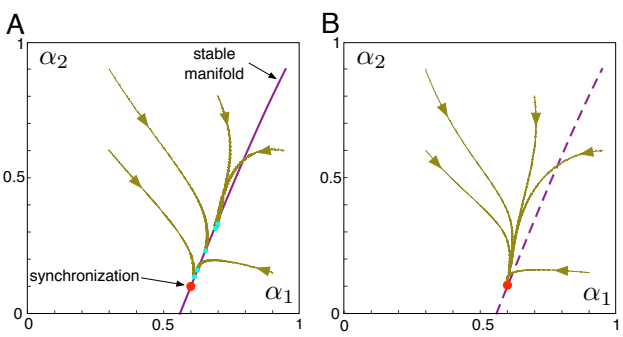

Fig. 2. Convergence of the learning process. Projection of trajectories of the system (5) to the plane (α_1, α_2) is shown. A) For $\tau = 0$ there exists a globally stable manifold (purple curve) that attracts all trajectories (cyan circles mark final points). Phase synchronization corresponds to the red point. B) For $\tau = 18$ the manifold disappears and all trajectories converge to the red point that warrants synchronization.

Introducing memory in the learning rule ($\tau > 0$), we can achieve exact replication of the coupling structure by the learner. For intermediate values of the memory constant, the previously mentioned manifold is destroyed and the learning process converges to the coupling structure of the teacher (Selskii and Makarov, 2016). Figure 2B illustrates numerical experiments similar to those discussed above, but now for $\tau = 18$. We observe convergence of the coupling strengths to α of the teacher. Consequently, phase synchronization of oscillations in the learner with the teacher is achieved.

4. REPLICATION OF COUPLING STRUCTURE IN A CHAIN

In Sect. 3 we discussed synchronization in a chain of two network motifs. Those results can be extended on a chain with arbitrary number of motifs.

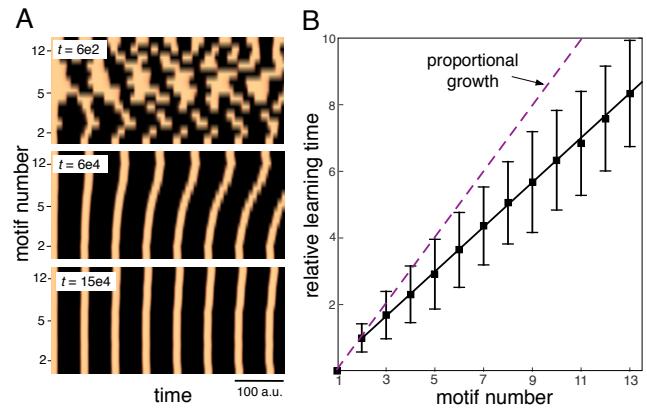

Fig. 3. Replication of motifs in a chain. A) Short epochs of oscillations of the first neuron along the chain of motifs (color corresponds to the value of $x_1(t)$): at the beginning (top), after some time (middle) and at the end (bottom) of learning. B) Relative time of learning along the chain T_n/T_1

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Figure 3A shows a representative example of learning in a chain of 13 motifs (to facilitate visual inspection in each panel we set initial phases of all motifs to zero). At the beginning, we set arbitrary $\alpha_n(0)$, $n > 1$, from uniform distribution, and hence we observe completely asynchronous oscillations in the chain (Fig. 3A, top). Then, due to learning the coupling strengths converge in a chain to the teacher (first motif) $\alpha_{13} \to \alpha_{12} \to \ldots \to \alpha_2 \to \alpha_1$ (Fig. 3A, middle and bottom).

Thus, for intermediate values of the memory constant, τ, all motifs are able to replicate the connectivity structure of the teacher. Moreover, the learning occurs gradually, i.e. more distal motifs require longer time to synchronize their activity with the teacher.

To estimate the synchronization time we performed a Monte Carlo test repeating simulations 20 times with arbitrary initial conditions $\alpha_n(0)$, $n = 2, 3, \ldots, 13$. Figure 3B shows the mean relative learning time, i.e. the ratio of the learning time of the n-th motif to the learning time of the first learner. The learning time grows linearly following the low $T_{n+1} = T_n + bT_1$, $n = 2, 3, \ldots$, where $b = 0.67$ is the growth ratio. Note that starting from the second motif the replication time is equal to 2/3 of the learning time required by the first learner.

5. CONSENSUS ON A RING

We now consider a chain closed into a ring, i.e. the last motif drives the first one (Fig. 1A). Then, there is no dedicated teacher and each network motif acts as a local teacher for the next motif in the ring. In this case, we have proven that independently on the initial conditions all motifs come to a state with the same connectivity pattern (Calvo et al., 2016). This final connectivity pattern corresponds to the mean initial values averaged over all network motifs:

$$\tilde{\alpha} = \frac{1}{N}\sum_{n=1}^{N} \alpha_n(0)$$

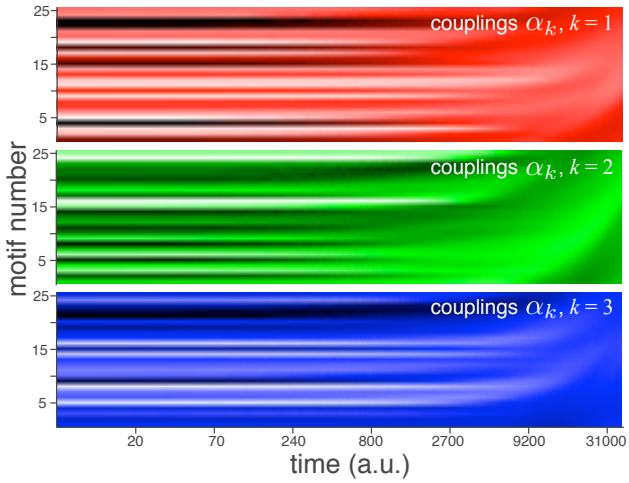

Fig. 4. Consensus behavior in a ring of 25 network motifs (red, green, and blue colors code the couplings of the corresponding neurons in different motifs).

Figure 4 illustrates the phenomenon of self-organization in a learning ring of 25 network motifs. Initially the connectivity patterns in all motifs were set random. Then, the learning without leader led to a consensus.

6. CONCLUSIONS

In this work we have proposed a model for learning heteroclinic circuits in networks of neural networks. The model is composed of a teacher and learner networks. At the beginning all networks implement winnerless competition dynamics through wandering around heteroclinic circuits with arbitrary dominant periods. Then, the purpose of learning is to synchronize oscillations in the learner with the teacher by tuning the coupling strengths in the learners.

The information transfer between the networks is implemented through a learning rule that changes local couplings in the learner according to the dynamics of the teacher. Thus, no direct influence of the teacher to the state variables of the learner exists. Instead the learner just "observes" the teacher and tunes itself to replicate the teacher structure. Such a mechanism of synchronization differs significantly from much more common models of synchronization based on couplings among state variables describing the system dynamics (Abarbanel et al., 1996).

The learning rule proposed in our model includes an integral operator, which implements a memory effect during the learning. We have shown that in the absence of memory the process of learning converges to a point in a stable one-parametric manifold. Existence of the attractive manifold leads to synchronization failure. Nevertheless, for intermediate values of the memory length the manifold can be destroyed and only a single equilibrium point survives, which ensures synchronization.

We have also shown that under appropriate training a chain of learner motifs can progressively "copy" the connectivity pattern of the teacher network motif. In average the time interval required for pattern replication grows linearly with the chain size. Thus, the learning process does not blow up and at the end we observe phase synchronized oscillations along the whole chain of network motifs. Besides, we have shown that in a learning chain closed into a ring the network motifs always come to a "consensus", i.e. to a state with the same connectivity pattern obtained by averaging the initial patterns of all network motifs in the ring. We foresee that the reported mechanism of learning can be useful for replication of scenarios of cognitive navigation in dynamic environments (Villacorta-Atienza and Makarov, 2013).

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